

参考答案

数学模拟试卷(一)

一、1. D 2. C 3. B 4. D 5. B 6. C 7. C 8. A 9. A 10. B

二、11. $x > \frac{9}{4}$ 12. 0.4 13. $\pm\sqrt{3}$ 14. ± 1 15. $-4, \frac{4}{9}$ 16. $2b = a + c$ ($a = b = c$ 或 $a = b + 1 = c + 2$ 各给 1 分)

三、17. (1) -1 ; (2) $4m^2 - 16n^2 = 4(m^2 - 4n^2) = 4(m + 2n)(m - 2n)$.

18. (1) $S_5 = -\frac{12b^5}{(a-1)^6}$; (2) $S_3 + S_4 = \frac{6b^2}{(a-1)^3} - \frac{8b^3}{(a-1)^4} = \frac{2b^2[3(a-1) - 4b]}{(a-1)^4} = \frac{2b^2(3a-4b-3)}{(a-1)^4}$. 当 $3a-4b=3$ 时, 原式 $= \frac{2b^2(3a-4b-3)}{(a-1)^4} = 0$.

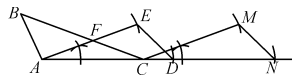
19. (1)

和的绝对值	-2	-1	1	3
-2	/	3	1	1
-1	3	/	0	2
1	1	0	/	4
3	1	2	4	/

任取两个数, 其和的绝对值有 0, 1, 2, 3, 4. 所以 k 的所有取值为 0, 1, 2, 3, 4; (2) 从表中可以得到: $P_1 = \frac{4}{12} =$

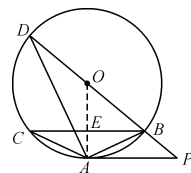
$$\frac{1}{3}, P_4 = \frac{2}{12} = \frac{1}{6}.$$

20. 解: (1) 证明: $\because FA = FC, \therefore \angle FAC = \angle FCA$. 在 $\triangle ABC$ 与 $\triangle EDA$ 中, $\angle FAC = \angle FCA, \angle D = \angle B, AD = BC, \therefore \triangle ABC \cong \triangle EDA$; (2) 如答图, $\triangle CMN$ 即为所求的三角形; (3) 将 $\triangle EDA$ 沿 AD 方向平移 5 cm 得 $\triangle CMN$, 再以点 C 为旋转中心, 逆时针方向旋转 160° 与 $\triangle ABC$ 重合.



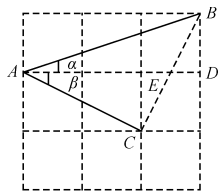
(第 20 题)

21. (1) 如答图, 连结 $AO, \because AB = AC, \therefore OA \perp BC$. 又 $\because PA \parallel BC, \therefore PA \perp AO$; (2) 设 OA 与 BC 交于点 $E, \because AB = AC, \therefore OA$ 平分 BC , 而 $BC = 4, \therefore BE = 2$. 又 $\because \tan \angle ABC = \frac{1}{2}, \therefore AE = 1$. 设 $OE = x, \therefore OB^2 = OE^2 + BE^2$, 即 $(1+x)^2 = x^2 + 2^2$, 解得 $x = \frac{3}{2}, \therefore BO = \frac{5}{2}, \therefore BD = 5$. $\because BD$ 为 $\odot O$ 的直径, $\therefore \triangle ABD$ 为直角三角形, 则 $AB^2 + AD^2 = BD^2, \therefore AD = \sqrt{BD^2 - AB^2} = \sqrt{5^2 - (\sqrt{5})^2} = 2\sqrt{5}$.

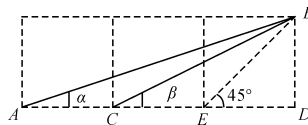


(第 21 题)

22. 解: 设正方形边长为 1. (1) 证明: 方法一: 如图①, 连结 $BC, \because AC = \sqrt{5}, AB = \sqrt{10}, BC = \sqrt{5}, \therefore AC^2 + BC^2 = AB^2, \therefore \angle ACB = 90^\circ, \therefore AC = BC, \therefore \triangle ABC$ 为等腰直角三角形, $\therefore \angle BAC = 45^\circ, \therefore \alpha + \beta = 45^\circ$.



①

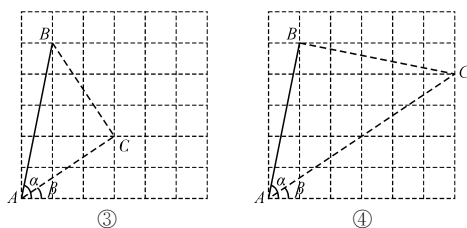


②

第 22 题图

方法二: 如图②, 连结 $BE, \because AE = 2, BE = \sqrt{2}, AB = \sqrt{10}$, 又 $\because CE = 1, BE = \sqrt{2}, BC = \sqrt{5}, \therefore \frac{AE}{BE} = \frac{AB}{BC} = \frac{BE}{CE}, \therefore$

$\triangle AEB \sim \triangle BEC, \therefore \angle CBE = \angle BAE = \alpha, \therefore \alpha + \beta = 45^\circ$; (2) 方法一: 如图③构图, $\because AB = \sqrt{26}, AC = \sqrt{13}, BC = \sqrt{13}, \therefore AC^2 + BC^2 = AB^2$, 又 $\because BC = AC, \therefore \angle CAB = 45^\circ$, 即 $\alpha - \beta = 45^\circ, \angle BAC$ 就是要画的角, 且这个角的度数为 45° .

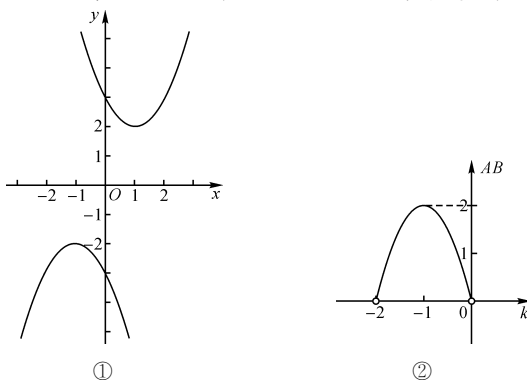


第 22 题图

方法二:如图④构图, $\because AB = \sqrt{26}, AC = \sqrt{52}, BC = \sqrt{26} \therefore AB^2 + BC^2 = AC^2$, 又 $\because BC = BA$, $\therefore \angle CAB = 45^\circ$, 即 $\alpha - \beta = 45^\circ$, $\angle BAC$ 就是要画的角, 且这个角的度数为 45° .

23. 解: (1) 当 $k=1$ 时, $y_1 = (x-1)^2 + 2, y_2 = -(x+1)^2 - 2$, 作图如图①所示, y_1, y_2 的图象关于原点成中心对称;

(2) $A(0, k^2 + 2k), B(0, -k^2 - 2k)$, 当 $-2 < k < 0$ 时, $AB = -2k^2 - 4k$, 如图②可知: $0 < AB \leq 2$;



第 23 题图

(3) ① $\because OA = OB, OC = OD$, 但是当 $k = -2$ 时, $OA = OB = 0$, 即 $AB = 0$, A 与 B 重合, \therefore 当 $k \neq -2$ 时, 四边形 $ACBD$ 是平行四边形; ② \because 对角线与 CD 不垂直, \therefore 四边形 $ACBD$ 不是菱形; ③ 当 $AB = CD$ 时, 四边形 $ACBD$ 是矩形, 有 $(2k^2 + 4k)^2 = 20k^2, k = -2 \pm \sqrt{5}$.

数学模拟试卷(二)

一、1. C 2. D 3. D 4. A 5. B 6. D 7. C 8. D 9. B 10. A

二、11. $\frac{1}{3}$ 12. 0 13. $\sqrt{5}, \frac{4}{5}$ 14. 2 0 16 15. $P(2, 4 - 2\sqrt{2}), Q(\sqrt{2}, \sqrt{2})$ 16. ①③

三、17. 解: (1) 众数是 9 度, 中位数是 9 度; (2) $(9 \times 3 + 10 \times 1 + 11 \times 1) \div 5 = 9.6$ (度), $9.6 \times 36 \times 22 = 7\ 603.2$ (度), 答: 估计该校该月的总用电量为 7 603.2 度.

18. 解: (1) $3 \times 7 = \frac{3}{7}$; (2) 略.

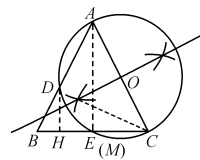
19. 证明: (1) $\because AB^2 = 5, BC^2 = 20, AC^2 = 25, \therefore AB^2 + BC^2 = AC^2, \therefore \triangle ABC$ 是直角三角形, $\angle ABC = 90^\circ$; (2) 在 $\text{Rt}\triangle ABC$ 中, $\because \tan C = \frac{AB}{AC} = \frac{\sqrt{5}}{5}$ 而 $\tan 30^\circ = \frac{\sqrt{3}}{3}, \therefore \angle C \neq 30^\circ$.

20. 解: (1) 证明: $\because E$ 是 AD 的中点, $\therefore AE = DE, \because AF \parallel BD, \therefore \angle FAE = \angle BDE, \angle AFE = \angle DBE, \therefore \triangle AFE \cong \triangle DBE, \therefore AF = BD, \because AF = DC, \therefore BD = DC$, 即 D 是 BC 的中点; (2) 四边形 $ADCF$ 是矩形时, $AB = AC$. 理由: $\because AF = DC, AF \parallel DC, \therefore$ 四边形 $ADCF$ 是平行四边形. $\because AB = AC, BD = DC, \therefore AD \perp BC$ 即 $\angle ADC = 90^\circ, \therefore$ 平行四边形 $ADCF$ 是矩形.

21. 解: (1) 如图, 作 $AM \perp BC$ 交 BC 于 $M. \because AC = 4\sqrt{5}, \sin C = \frac{2\sqrt{5}}{5}, \therefore BC = 2MC =$

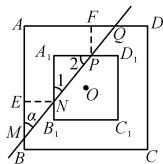
$2\sqrt{(4\sqrt{5})^2 - 8^2} = 8$; (2) 如图所示, $\odot O$ 即为所求. 作 $DH \perp BC$ 于点 H , 连结 CD 得 CD

$\perp AD$, 设 $BD = a, AD = 4\sqrt{5} - a$, 由勾股定理得 $AC^2 - AD^2 = BC^2 - BD^2$, 即 $(4\sqrt{5})^2 - (4\sqrt{5} - a)^2 = 8^2 - a^2$, 解得 $a = \frac{8\sqrt{5}}{5}, \therefore \sin B = \sin \angle ACB = \frac{2\sqrt{5}}{5} = \frac{DH}{a}, \therefore DH = \frac{16}{5}$.



(第 21 题)

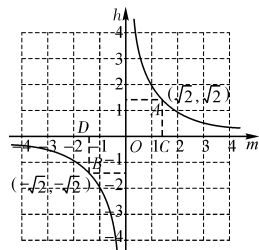
22. 解:如图,作 $NE \perp AB$ 于 E , $PF \perp AD$ 于 F . 由题意得 $NE = PF$, 设 $NE = PF = x$. (1) 在 $\text{Rt} \triangle MNE$ 和 $\text{Rt} \triangle PQF$ 中, $\because \alpha$ 等于 30° , $\therefore MN = 2x$, 而 $PQ = \frac{2x}{\sqrt{3}} = \frac{2\sqrt{3}}{3}x$, 则 $\frac{MN}{PQ} = \frac{\sqrt{3}}{1}$; (2) 在 $\text{Rt} \triangle MNE$ 中, $\angle EMN = \alpha$, $\sin \alpha = \frac{EN}{MN} = \frac{x}{MN}$, 得 $MN = \frac{x}{\sin \alpha}$, 在 $\text{Rt} \triangle AMQ$ 中, $\angle AQM = 90^\circ - \alpha$, \therefore 在 $\text{Rt} \triangle PQF$ 中, $\sin \angle FQP = \sin(90^\circ - \alpha) = \frac{PF}{PQ} = \frac{x}{PQ}$, 得 $PQ = \frac{x}{\sin(90^\circ - \alpha)}$, $\therefore \frac{MN}{PQ} = \frac{x}{\sin \alpha} \div \frac{x}{\sin(90^\circ - \alpha)} = \frac{\sin(90^\circ - \alpha)}{\sin \alpha}$.



(第 22 题)

23. 解:(1) 证明:在二次函数 $y = x^2 - (2m-1)x + m^2 - m$ 中, $\because \Delta = 1 > 0$, \therefore 不论 m 取何值时, 该二次函数图象总与 x 轴有两个交点; (2) 由点 $A(n-3, n^2+2)$ 与点 $B(-n+1, n^2+2)$ 的坐标可知二次函数的对称轴为直线 $x = \frac{(n-3) + (-n+1)}{2} = -1$, 由二次函数的表达式可知对称轴为直线 $x = -\frac{-(2m-1)}{2}$, $\therefore -\frac{-(2m-1)}{2} = -1$, 得 $m = -\frac{1}{2}$, 可知函数表达式为 $y = x^2 + 2x + \frac{3}{4}$, 将 $(n-3, n^2+2)$ 代入函数表达式得 $n = \frac{7}{16}$. \therefore 二次函数表达式为 $y = x^2 + 2x + \frac{3}{4}$, $n = \frac{7}{16}$;

(3) 由二次函数 $y = x^2 - (2m-1)x + m^2 - m$ 分解因式可得 $y = (x-m)[x-(m-1)]$, 即图象与 x 轴两个交点的横坐标分别为 $x_1 = m, x_2 = m-1$ (其中 $x_1 > x_2$), (也可以用求根公式求得方程的两根). h 是关于 m 的函数, 且 $h = 2 - \frac{2x_2}{x_1}$, $\therefore h = 2 - \frac{2(m-1)}{m} = 2 - \frac{2m-2}{m} = 2 - (2 - \frac{2}{m}) = \frac{2}{m}$ (其中 m 是常数, 且 $m \neq 0$), 作出此函数的图象如图, 当 $h = m$ 时, 有 $m = \frac{2}{m}$, 解得 $m = \pm\sqrt{2}$, 从图上可以看出在垂线 AC 的右侧和垂线 BD 与 h 轴之间时有 $h < m$, \therefore 当 $h < m$ 时, $-\sqrt{2} < m < 0$ 或 $m > \sqrt{2}$.



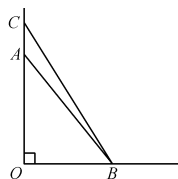
(第 23 题)

数学模拟试卷(三)

一、1. A 2. B 3. A 4. B 5. A 6. D 7. D 8. A 9. B 10. B

二、11. -4 12. $\sqrt{13}$ 13. -4 或 0 14. 102° 15. 24 或 $\sqrt{26}$ 或 $5\sqrt{26}$ 16. $\frac{\sqrt{3}}{8}; \frac{\sqrt{3}}{2^n}$

三、17. 解:如图,形如 $\triangle AOB$ 和 $\triangle COB$ 的两种图形:在 $\triangle AOB$ 中,直角边 $OB = a$,斜边 $AB = b$; 在 $\triangle COB$ 中,直角边 $OB = a$,直角边 $CO = b$.



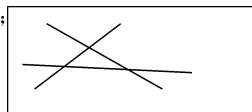
(第 17 题)

18. 解:不等式 $2x+a > 3$ 的解为 $x > \frac{3-a}{2}$, 不等式 $5x-b < 2$ 的解为 $x < \frac{b+2}{5}$, 不等式组的解为

$$-1 < x < 1, \text{由题意得} \begin{cases} \frac{3-a}{2} = -1, \\ \frac{b+2}{5} = 1, \end{cases} \text{解得} \begin{cases} a = 5, \\ b = 3, \end{cases} \therefore ab = 15.$$

19. 解:(1) 落在 $y_1 = x-3$ 图象上的点有: $A(0, -3), B(3, 0), C(-1, -4)$, 落在 $y_2 = x^2 - 2x - 3$ 图象上的点有 $A(0, -3), B(3, 0)$, $\therefore P(1) = \frac{2}{3}$; (2) $\because A(0, -3), B(3, 0), C(-1, -4)$ 中同时在该抛物线上的只有 A, B , $\therefore P(2) = \frac{1}{3}$.

20. 解:(1) 如图:

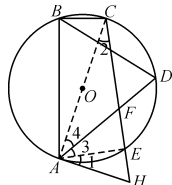


(第 20 题)

(2) 最多 11 部分; (3) $a_{n+1} - a_n = n + 1$.

21. 解:(1) $\because \angle ACB = 90^\circ, CH \perp BM, \therefore \angle MCH = \angle CBM, \therefore M$ 是边 AC 的中点, \therefore 设 $AM = CM = x$, 则 $BC = AC = 2x$, 在 $\text{Rt} \triangle BCM$ 中, $BM = \sqrt{5}x, \therefore \sin \angle MCH = \sin \angle CBM = \frac{MC}{BM} = \frac{x}{\sqrt{5}x} = \frac{\sqrt{5}}{5}$; (2) 证明: $\because \angle ACB = 90^\circ, CH \perp BM, \therefore \angle MHC = \angle MCB, \angle MCH = \angle MBC, \therefore \triangle MCH \sim \triangle MBC$; (3) $\because \triangle MCH \sim \triangle MBC, \therefore MC^2 = MH \cdot MB, \therefore AM^2 = MH \cdot MB, \therefore \triangle MAH \sim \triangle MBA, \therefore \angle AHM = \angle BAM = 45^\circ$.

22. 解:(1)证明:如图,连结 AC 和 AE , $\because AB \perp BC$, $\therefore AC$ 是 \odot 的直径, $\therefore \angle D = \angle ACB$, $\therefore \tan D = \tan \angle ACB = 3$, 在 $Rt\triangle ABC$ 中, $BC = 2$, $\therefore AB = 3BC = 6$, 由勾股定理, 得 $AC = 2\sqrt{10}$, 在 $\triangle CAH$ 中, 由勾股定理逆定理, 得 $AC^2 + AH^2 = 50 = CH^2$. $\therefore \angle CAH = 90^\circ$, 即 $CA \perp AH$, $\therefore AH$ 是 \odot 的切线; (2)证明: $\because D$ 是 \widehat{CE} 的中点, $\therefore \angle 3 = \angle 4$, $\because AC$ 是 \odot 的直径, $\therefore AE \perp CH$, $\angle H + \angle 1 = \angle H + \angle 2 = 90^\circ$, $\angle 1 = \angle 2$, $\therefore \angle 1 + \angle 3 = \angle 2 + \angle 4$, 而 $\angle AFH = \angle 3 + \angle 4$, 即 $\angle AFH = \angle HAF$, $\therefore HF = HA$;
(3) $\because HF = HA$, $\therefore HF = HA = \sqrt{10}$, $\because CA \perp AH, AE \perp CH$, $\therefore AH^2 = EH \cdot CH$, 得 $EH = \sqrt{2}$, $\therefore EF = \sqrt{10} - \sqrt{2}$.



23. 解:(1)直线 AC 的表达式为 $y = -2x - 2$; (2)点 D 的坐标为 $(-\frac{1}{2}, -1)$, $S_{\triangle BAD}$ 的面积 (第 22 题) 为 $\frac{5}{4}$; (3)借助 $\triangle BAD$ 的一边构造与 $\triangle BAD$ 面积相等的三角形, 有 5 个点: $P_1(\frac{3}{2}, \frac{5}{4}), P_2(\frac{3}{2}, -\frac{5}{4}), P_3(\frac{3}{2}, 1), P_4(\frac{3}{2}, -1), P_5(\frac{3}{2}, -10)$.

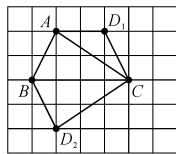
数学模拟试卷(四)

一、1. A 2. C 3. B 4. C 5. C 6. D 7. B 8. A 9. D 10. A

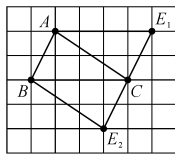
二、11. $>$ 12. 24° 13. $(x+y)(x-y)$ 14. 2 15. $2k$ 16. $2 + \sqrt{3}, 2 - \sqrt{3}$ 或 $\frac{\sqrt{3}}{3}$

三、17. 解: $\because x^2 - y^2 = 3xy$, \therefore 原式 $= \frac{(x^2 - y^2) - xy}{2(x^2 - y^2) + xy} = \frac{3xy - xy}{2 \cdot 3xy + xy} = \frac{2xy}{7xy} = \frac{2}{7} (xy \neq 0)$.

18. 解:如图.



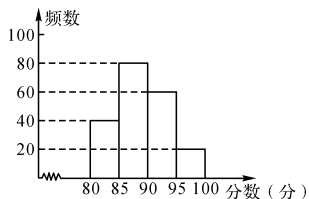
①



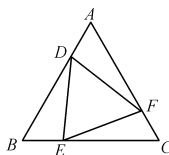
②

(第 18 题)

19. 解:(1)总人数为 $20 \div 0.1 = 200$ (人), $a = 200 \times 0.2 = 40$ (人), $b = 80 \div 200 = 0.4$, $c = 60 \div 200 = 0.3$, 补全直方图如图所示; (2)中位数落在 $85 \leq x < 90$ 这一段; (3)平均分为 $(82.5 \times 40 + 87.5 \times 80 + 92.5 \times 60 + 97.5 \times 20) \div 200 = 89$ (分).



(第 19 题)



(第 20 题)

20. 解:(1)证明: $\because \triangle ABC$ 是等边三角形, $\therefore \angle A = \angle B = \angle C = 60^\circ, AB = AC = BC$, $\therefore AD = BE = CF$, $\therefore AF = BD = CE$, $\therefore \triangle ADF \cong \triangle BED \cong \triangle CFE$ (SAS), $\therefore DF = DE = EF$, 即 $\triangle DEF$ 是等边三角形; (2) $\because \triangle ABC, \triangle DEF$ 都是等边三角形, $\therefore \triangle ABC \sim \triangle DEF$. 若 $\angle FEC = 90^\circ$, 可得 $EF = \sqrt{3} EC = \frac{\sqrt{3}}{3} AC$, $\therefore \frac{S_{\triangle DEF}}{S_{\triangle ABC}} = \frac{1}{3}$, 若 $\angle EFC = 90^\circ$, 可得 $EF = \sqrt{3} FC = \frac{\sqrt{3}}{3} AC$, $\therefore \frac{S_{\triangle DEF}}{S_{\triangle ABC}} = \frac{1}{3}$.

21. (1)由题意得 $3 < 2x - 1 < 9$, $\therefore 2 < x < 5$. $\because x$ 为整数, $\therefore x$ 的值为 3 或 4, \therefore 符合条件的数对为 $(3, 6, 5), (3, 6, 7)$;

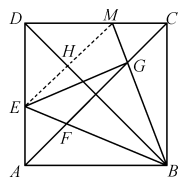
(2)作边长为 3, 6, 5 的三角形(图略), 此三角形内切圆半径为 $\frac{2\sqrt{14}}{7}$.

22. 解:证明:(1)将 $x = -2$ 代入, 得 $y = k \cdot (-2)^2 + (2k - 1) \cdot (-2) - 2 = 0$, \therefore 不论 k 取何值, 此函数图象一定经过点 $(-2, 0)$; (2)①若 $k = 0$, 此函数为一次函数 $y = -x - 2$, 当 $x > 0$ 时, y 随 x 的增大而减小, $\therefore k = 0$ 符合题意

②若 $k \neq 0$, 此函数为二次函数, 而图象一定经过 $(-2, 0), (0, 2)$, \therefore 要使当 $x > 0$ 时, y 随 x 的增大而减小, 须满足 $k < 0$ 且 $-\frac{2k-1}{2k} = \frac{1}{2k} - 1 < 0$, $\therefore k < 0$. 综上, k 的取值范围是 $k \leq 0$; (3)若 $k = 0$, 此函数为一次函数 $y =$

$-x-2$, $\therefore x$ 的取值为全体实数, $\therefore y$ 无最小值, 若 $k \neq 0$, 此函数为二次函数, 若存在最小值为 -3 , 则 $\frac{-8k-(2k-1)^2}{4k} = -3$ 且 $k > 0$, 解得 $k = \frac{2 \pm \sqrt{3}}{2}$, 符合题意.

23. 解: (1) \because 四边形 $ABCD$ 是正方形, $\therefore \angle BDE = \angle BCG = \angle CBD = 45^\circ$, $BD = \sqrt{2}BC$, 又 $\because \angle EBM = 45^\circ$, $\therefore \angle DBE = \angle CBG$, $\therefore \triangle BDE \sim \triangle BCG$, $\therefore DE : CG = BD : BC = \sqrt{2} : 1$; (2) ① \because 四边形 $ABCD$ 是正方形, 且 $\triangle BDE \sim \triangle BCG$, $\therefore BE : BG = BD : BC = \sqrt{2} : 1$, $\therefore \triangle BEG \sim \triangle BDA$, $\therefore \triangle BEG$ 是等腰直角三角形, $\therefore y = S_{\triangle BEG} = \frac{1}{4}(AE^2 + AB^2) = \frac{1}{4}BE^2 = \frac{1}{4}x^2 + 9$ ($0 < x < 6$); ② 若 E, M 关于对角线 BD 成轴对称, 如图, 连结 EM 交 AC 于点 H , $\therefore BD$ 垂直平分 EM , BE 平分 $\angle ABD$, $\therefore AE = HE = DH$, $DE = \sqrt{2}HE$, $\therefore 6 - x = \sqrt{2}x$, 即 $x = 6\sqrt{2} - 6$, $\therefore y = \frac{1}{4} \times (6\sqrt{2} - 6)^2 + 9 = 36 - 18\sqrt{2}$.



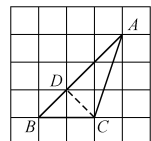
(第 23 题)

数学模拟试卷(五)

一、1. B 2. C 3. A 4. C 5. B 6. B 7. A 8. A 9. D 10. A

二、11. $\pi - 1$ 12. $a + 1$ 13. $\frac{2}{3}$ 14. $x_1 > x_2$ -1 15. $6\pi + 18$ 16. $(4 + m, \frac{5}{3})$ 或 $(4 + m, \frac{13}{3})$

三、17. 解: (1) $\cos B = \cos 45^\circ = \frac{\sqrt{2}}{2}$, $\tan(\angle ACB - 90^\circ) = \frac{1}{3}$; (2) 如图, 作 CD . 易知 $CD \perp AB$, $\therefore CD = \sqrt{2}$, $AC = \sqrt{10}$, $\therefore \sin A = \frac{\sqrt{2}}{\sqrt{10}} = \frac{\sqrt{5}}{5}$.



(第 17 题)

18. 解: (1) $\because 0 < \alpha < 90^\circ, 0 < \beta < 90^\circ, 90^\circ < \gamma < 180^\circ$, $\therefore 90^\circ < \alpha + \beta + \gamma < 360^\circ, 30^\circ < \frac{1}{3}(\alpha + \beta + \gamma) < 120^\circ$, $\therefore \alpha + \beta + \gamma = 3 \times 119^\circ = 357^\circ$; (2) 由题意得

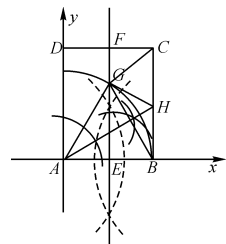
$$\begin{cases} \alpha + \beta + \gamma = 357, \\ \beta = \alpha - 1, \\ \gamma = 2\alpha, \end{cases} \quad \text{解得 } \alpha = 89.5^\circ.$$

19. 解: (1) $5 \div 10\% = 50$ (户); (2) $50 - 10 - 5 = 35$ (户), B 类的户数: $35 \times \frac{3}{3+4} = 15$ (户), C 类: $35 - 15 = 20$ (户);

(3) 图略, $\frac{20}{50} \times 360^\circ = 144^\circ$ (4) $1500 \times \frac{10}{50} = 300$ (户)

20. 解: $\because m > 0, n > 0, m \neq n$, $\therefore (m - n)^2 > 0$, 即 $m^2 + n^2 > 2mn$, $\therefore b - a = \frac{1}{2}(m^2 + n^2) - mn = \frac{1}{2}(m - n)^2 > 0$, 即 $b > a$, $b - c = \frac{1}{2}(m^2 + n^2) - \frac{1}{4}(m - n)^2 = \frac{1}{4}(2m^2 + 2n^2 - m^2 - n^2 + 2mn) = \frac{1}{4}(m + n)^2 > 0$, 即 $b > c$, $\therefore b$ 是三边中的最大边. \because 又 $a + c = mn + \frac{1}{4}(m - n)^2 = \frac{1}{4}(m^2 + n^2 + 2mn) < \frac{1}{4}(2m^2 + 2n^2) = \frac{1}{2}(m^2 + n^2) = b$, $\therefore a, b, c$ 三条线段不能组成三角形.

21. 解: (1) 如右图; (2) 证明 $\because EF$ 是 AB 的中垂线, $\therefore AG = BG$, 由折叠的性质得 $AG = AB$, $\therefore AB = AG = BG$, $\therefore \triangle ABG$ 是等边三角形; (3) 方法一: $\because GH = BH = \frac{\sqrt{3}}{3} \times 3\sqrt{3} = 3$, $CH = BC - BH = m - 3$, $FG = EF - GE = m - \frac{9}{2}$, 若折叠后点 A, G, C 在同一直线上, 则须 $\angle HGC = 90^\circ$, 即 $CG^2 + GH^2 = CH^2$, $\therefore (\frac{3\sqrt{3}}{2})^2 + (m - \frac{9}{2})^2 + 9 = (m - 3)^2$, 解得 $\therefore m = 9$. 方法二: 如图建立平面直角坐标系, 则点 G 坐标为 $(\frac{3\sqrt{3}}{2}, \frac{9}{2})$, 点 C 坐标为 $(3\sqrt{3}, m)$, \therefore 直线 AG 的函数解析式为 $y = \sqrt{3}x$, \therefore 由题意得要使点 C 在直线 AG 上, 则有 $m = 3\sqrt{3} \times \sqrt{3} = 9$.



(第 21 题)

22. 解: (1) $a = -1$, 顶点坐标为 $(\frac{1}{2}, \frac{9}{4})$; (2) 由题意得 $\begin{cases} a > 0 \\ -\frac{1}{2a} < 0, \text{ 解得 } 0 < a < \frac{1}{8}; \\ 1 - 8a > 0, \end{cases}$ (2) 方法一: ① 显然 $a_1 \neq$

$a_2 \neq 0$, ②若 $a > 0$, 则抛物线对称轴为直线 $x = -\frac{1}{2a} < 0$, 而抛物线与 y 轴正半轴交于点 $(0, 2)$, 所以抛物线与 x 轴要么没有交点, 要么交点均在 x 轴负半轴, 此与题目条件矛盾, $\therefore a_1, a_2$ 不可能为正数, ③ $\because a_1 < 0, a_2 < 0$, 抛物线又过定点 $(0, 2)$ 且 $n > m > 0$, $\therefore -\frac{1}{2a_1} < -\frac{1}{2a_2}$, 即 $\frac{1}{2a_1} > \frac{1}{2a_2}$, $\therefore a_2 > a_1$. 方法二: $\because a_1 m^2 + m + 2 = 0 \quad a_2 n^2 + n + 2 = 0, m < n$, $\therefore a_1 - a_2 = -\frac{m+2}{m^2} + \frac{n+2}{n^2} = \frac{(n+2) \cdot m^2 - (m+2) \cdot n^2}{m^2 \cdot n^2} = \frac{(m-n)(mn+2m+2n)}{m^2 \cdot n^2} < 0$, $\therefore a_1 < a_2$.

23. 解: 如图, 作 $DE \perp Ox$ 于 E . (1) $\because AB \perp Ox, DE \perp Ox, \therefore AB \parallel DE, \therefore \triangle CAB \sim \triangle CED, \therefore \frac{AB}{DE} = \frac{BC}{CD} = \frac{1}{2}$. $\because AB = 2, \therefore DE = 4$, 即点 D 到射线 Ox 的距离为 4;

(2) $\because \frac{AB}{DE} = \frac{BC}{CD}, \therefore \frac{2}{DE} = \frac{1}{k}, \therefore DE = 2k. \because \frac{DE}{OE} = \tan \angle xOy = \tan \angle ABO = \frac{OA}{OB} = \frac{4}{2}, \therefore OE = \frac{1}{2} DE = k, \therefore AE = 4 - k. \because \triangle CAB \sim \triangle CED, \therefore \frac{CA}{CE} = \frac{BC}{CD}, \therefore$

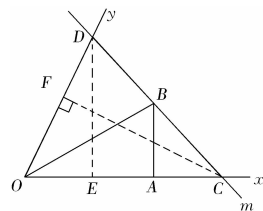
$\frac{CA}{CA+4-k} = \frac{1}{k}, \therefore CA = \frac{4-k}{k-1}, \therefore OC = \frac{4-k}{k-1} + 4 = \frac{3k}{k-1}. \therefore S_{\triangle OCD} = \frac{1}{2} OC \cdot DE =$

$\frac{1}{2} \cdot \frac{3k}{k-1} \cdot 2k = \frac{3k^2}{k-1} (1 < k \leq 4)$; (3) 如图, ①当 $OC = CD$ 时, 过 C 作 $CF \perp Oy$ 于 F , 则 $OF = \frac{1}{2} OD = \frac{1}{2}$

$\sqrt{k^2 + (2k)^2} = \frac{\sqrt{5}}{2} k. \therefore \frac{CF}{OF} = \tan \angle xOy = 2, \therefore CF = \sqrt{5} k. \therefore S_{\triangle OCD} = \frac{1}{2} OD \cdot CF, \therefore \frac{1}{2} \cdot \sqrt{5} k \cdot \sqrt{5} k = \frac{3k^2}{k-1}, \therefore k =$

$\frac{11}{5}$. ②当 $OD = CD$ 时, $OE = CE = \frac{1}{2} OC, \therefore k = \frac{1}{2} \cdot \frac{3k}{k-1}, \therefore k = \frac{5}{2}$. ③当 $OC = OD$ 时, $\sqrt{5} k = \frac{3k}{k-1}, \therefore k = \frac{3\sqrt{5}}{5} +$

1. 综上所述, 当 $\triangle OCD$ 是等腰三角形时, k 的值为 $\frac{11}{5}, \frac{5}{2}$ 或 $\frac{3\sqrt{5}}{5} + 1$.



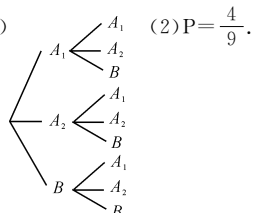
(第 23 题)

数学模拟试卷(六)

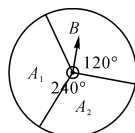
一、1. D 2. A 3. B 4. C 5. A 6. A 7. B 8. C 9. B 10. A

二、11. $x=0$ 或 $x=2$ 12. $\frac{12}{5}$ 13. 6 14. 5 15. 20 min 480 16. $3-\sqrt{3}$ 或 $\frac{12-2\sqrt{3}}{11}$

三、17. 解: 如图, 将 A 分割成相等的两部分 A_1, A_2 . (1)

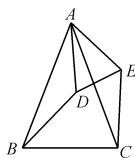


(2) $P = \frac{4}{9}$.



(第 17 题)

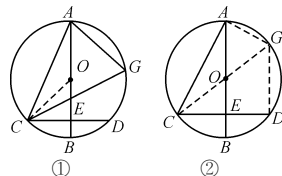
18. 解: (1) 作图, 如图所示; (2) 证明: $\because \angle DAE = \angle BAC, \therefore \angle BAD = \angle CAE$, 又 $\because AB = AC, AD = AE, \therefore \triangle ABD \cong \triangle ACE, \therefore BD = EC$.



(第 18 题)

19. 解: (1) $A = 8xy, B = y^2$; (2) 当 $y = 2$ 时, $16x > 4, x > \frac{1}{4}$.

20. 解: (1) \because 直径 $AB \perp CD, \therefore \widehat{AC} = \widehat{AD}, \therefore \angle ACD = \angle AGC$; (2) 证明: 如图②, $\because AB \parallel DG, AB \perp CD, \therefore DG \perp CD, \therefore CG$ 是直径, $\therefore \angle CAG = 90^\circ$, 从而 $\angle CAG = \angle AEC$, 又 $\because \angle ACD = \angle AGC, \therefore \triangle ACG \sim \triangle AEC$; (3) 如图①, 连结 $OC. \because OE = BE = \frac{1}{2} OB, \angle OEC = 90^\circ, \therefore \cos \angle COE = \frac{1}{2}, \therefore \angle COE = 60^\circ, \therefore \angle AGC =$



(第 20 题)

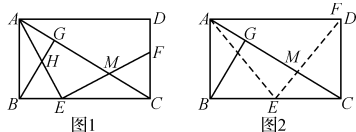
$\frac{1}{2} \angle AOC = \frac{1}{2} \times (180^\circ - 60^\circ) = 60^\circ$.

21. 解: (1) 易知 $A(-2, 0), B(4, 0)$, 故对称轴为直线 $x = 1$. \therefore 顶点坐标为 $P(1, -9a)$, 又 \because 点 P 在直线 $y = 3x + m$ 上, $\therefore 3 + m = -9a, \therefore a = -\frac{m+3}{9}$; (2) $\because \triangle ABP$ 为直角三角形, 故由对称性知 $\triangle ABP$ 为等腰直角三角形, \therefore

$|m+3| = 3, \therefore m_1 = 0, a_1 = -\frac{1}{3}$ 或 $m_2 = -6, a_2 = \frac{1}{3}, \therefore$ 函数表达式分别为 $y = -\frac{1}{3}(x+2)(x-4)$ 与 $y = 3x$,

或 $y = \frac{1}{3}(x+2)(x-4)$ 与 $y = 3x - 6$.

22. 解: (1) $\triangle ABH \sim \triangle ECM$. 证明: $\because \angle ABC = 90^\circ, BG \perp AC, \therefore \angle ABG = \angle ECM$, 又 $\because \angle AEM = 90^\circ, \therefore \angle AEB + \angle MEC = 90^\circ, \therefore \angle BAH = \angle MEC, \therefore \triangle ABH \sim \triangle ECM$; (2) ①由题可得点 F 与点 D 重合, 画图如图 2 所示. $\because \triangle ADM \sim \triangle CEM, AB = BE = CE = CD = 2, \therefore EM : DM = CE : AD = 1 : 2, \therefore EM : ED = 1 : 3, \therefore ED = 2\sqrt{2}, \therefore EM = \frac{2\sqrt{2}}{3}$; ② $AG : GM : MC = 3 : 7 : 5$.



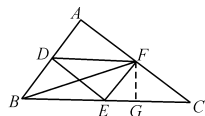
(第 22 题)

23. 解: (1) $\because k = \frac{1}{3}$ 即 $AD = \frac{1}{3}AB, BE = \frac{1}{3}BC, \therefore BD = \frac{2}{3}AB, \therefore \frac{S_{\triangle BDE}}{S_{\triangle ABC}} = \frac{\frac{1}{2}BD \cdot BE}{\frac{1}{2}BA \cdot BC} = \frac{BD}{BA} \cdot \frac{BE}{BC} = \frac{2}{9}$ (也可连结

AE 来求); (2) 由(1)的方法可求得 $\frac{S_{\triangle BDE}}{S_{\triangle BAC}} = \frac{\frac{1}{2}BD \cdot BE}{\frac{1}{2}BA \cdot BC} = \frac{BD}{BA} \cdot \frac{BE}{BC} = k(1-k)$,

同理: $\frac{S_{\triangle EFC}}{S_{\triangle BAC}} = k(1-k), \frac{S_{\triangle ADF}}{S_{\triangle BAC}} = k(1-k), \therefore m = \frac{S_{\triangle DEF}}{S_{\triangle BAC}} = 1 - \frac{S_{\triangle BDE}}{S_{\triangle BAC}} - \frac{S_{\triangle EFC}}{S_{\triangle BAC}} + \frac{S_{\triangle ADF}}{S_{\triangle BAC}} = 1 - 3k(1-k) = 3k^2 - 3k +$

1 ; (3) ① $\because m = 3k^2 - 3k + 1 = 3\left(k - \frac{1}{2}\right)^2 + \frac{1}{4}, \therefore$ 当 $k = \frac{1}{2}$ 时, m 取最小值, 此时四边形 $ADEF$ 是矩形; ② 如图, 由已知可设 $AB = 3a, AC = 4a, \therefore BC = 5a, BF = \sqrt{13}a, \therefore \sin \angle CBF = \frac{2S_{\triangle BFC}}{BF \cdot BC} = \frac{3a^2}{\sqrt{13}a \cdot 5a} = \frac{6\sqrt{13}}{65}$ (也可过点 F 作 $FG \perp BC$ 转化到 $\text{Rt}\triangle BFG$ 中来).



(第 23 题)

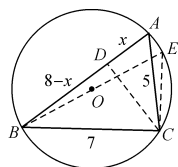
数学模拟试卷(七)

一、1. D 2. A 3. C 4. D 5. C 6. A 7. B 8. A 9. B 10. B

二、11. 20 12. $y = -\frac{1}{9}(x-6)^2 + 4$ 13. $\frac{\sqrt{2}}{4}$ 14. 20 15. $(0, \frac{5}{2})$ 16. $2\sqrt{2}$ 或 $4 - 2\sqrt{2}$

三、17. 解: $\sqrt{27} - 3\sin 60^\circ - \cos 30^\circ + 2\tan 45^\circ = 3\sqrt{3} - 3 \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} + 2 \times 1 = \sqrt{3} + 2$

18. 解: (1) 作图(略); (2) 作 $CD \perp AB$ 于 D . 设 $AD = x$, 则 $BD = 8 - x$. 由勾股定理, 得 $AC^2 - AD^2 = BC^2 - BD^2, \therefore 5^2 - x^2 = 7^2 - (8 - x)^2$, 解得 $x = \frac{5}{2}, \therefore CD = \sqrt{5^2 - (\frac{5}{2})^2} = \frac{5}{2}\sqrt{3}$. 作直径 BE , 连结 CE , 则 $\angle ECB = 90^\circ. \because \angle A = \angle E, \therefore \sin A = \sin E, \therefore \frac{CD}{AC} = \frac{BC}{BE}, \therefore BE = \frac{AC \cdot BC}{CD} = \frac{5 \times 7}{\frac{5}{2}\sqrt{3}} = \frac{14}{\sqrt{3}}. \therefore \triangle ABC$ 的外接圆半径为 $\frac{7}{\sqrt{3}}$.



(第 18 题)

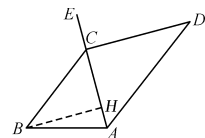
19. 解: (1) 顶点 $(1, 8)$, 对称轴是直线 $x = 1$, 与 y 轴的交点是 $(0, 6)$, 与 x 轴的交点是 $(3, 0), (-1, 0)$, 图象略; (2) 当 $x \leq 1$ 时, y 随 x 的增大而增大; 当 $x \geq 1$ 时, y 随 x 的增大而减小. 函数的最大值是 8.

20. 解: (1) 将 $x = 4, y = -1$ 代入方程, 得 $4 + a = 6$, 即 $a = 2$; (2) 列表得

$x \backslash y$	-1	0	2
-2	$(-2, -1)$	$(-2, 0)$	$(-2, 2)$
4	$(4, -1)$	$(4, 0)$	$(4, 2)$
6	$(6, -1)$	$(6, 0)$	$(6, 2)$

所有等可能的情况有 9 种, 其中 (x, y) 恰好为方程 $x - 2y = 6$ 的解的情况有 $(4, -1), (6, 0)$, 2 种, \therefore 甲、乙随机抽取一次的数恰好是方程 $x - ay = 6$ 的解的概率: $P = \frac{2}{9}$.

21. 解: (1) $\because AC \perp CD, \therefore \angle ACD = 90^\circ, \therefore AD = \sqrt{AC^2 + CD^2} = \sqrt{45^2 + 60^2} = 75.0$ (cm). 答: 车架档 AD 的长为 75.0 cm; (2) 过点 B 作 $BH \perp AC$ 于点 H , 如图所示. $\because AD \parallel$



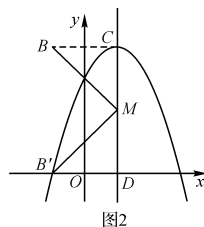
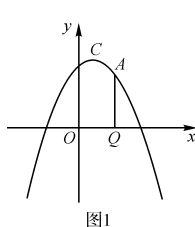
(第 21 题)

BC , $\therefore \angle BCA = \angle CAD$, $\therefore \tan \angle BCA = \tan \angle CAD$, 即 $\frac{BH}{CH} = \frac{CD}{CA} = \frac{60.0}{45.0} = \frac{4}{3}$. 设 $CH = 3k$, 则 $BH = 4k$, 在 $Rt \triangle ABH$ 中, $\therefore \tan \angle BAH = \frac{BH}{AH}$, $\therefore AH = \frac{BH}{\tan \angle BAH} = \frac{4k}{\tan 76^\circ} \approx \frac{4k}{4.0} = k$, $\therefore AC = AH + CH = k + 3k = 4k$, $\therefore BH = AC = 45.0 \text{ cm}$. $\therefore \sin \angle BAH = \frac{BH}{AB}$, $\therefore AB = \frac{BH}{\sin \angle BAH} = \frac{45.0}{\sin 76^\circ} \approx \frac{45.0}{0.97} \approx 46.4 \text{ cm}$. 答: 车链横档 AB 的长约为 46.4 cm .

22. 解: (1) 由题意, 得 $W = y(x - 20) = (-10x + 500)(x - 20)$, 整理, 得 $W = -10x^2 + 700x - 10\,000$, 即 $W = -10(x - 35)^2 + 2\,250 (20 \leq x \leq 50)$. $\therefore a = -10 < 0$, $\therefore W$ 有最大值, 且当 $x = 35$ (元) 时, $W_{\text{最大值}} = 2\,250$ (元). 答: 当销售单价定为每台 35 元时, 每月可获得最大利润, 最大利润为 $2\,250$ 元; (2) 当 $W = 2\,000$ 时, $-10x^2 + 700x - 10\,000 = 2\,000$, 整理, 得 $x^2 - 70x + 1200 = 0$, 解得 $x_1 = 30, x_2 = 40$. 答: 当销售单价定为 30 元/台或 40 元/台时, 每月可获得 $2\,000$ 元的利润; (3) $\because 20 \leq x \leq 50$, 且 $x \leq 32$, $\therefore x$ 的取值范围是 $20 \leq x \leq 32$. 在 $W = -10(x - 35)^2 + 2\,250$ 中, $\therefore a = -10 < 0$, \therefore 当 $20 \leq x \leq 32$ 时, W 随着 x 的增大而增大. \therefore 当 $W \geq 2\,000$ 时, x 的取值范围是 $30 \leq x \leq 32$, 而成本 $= 20y = 20 \times (-10x + 500)$ 随 x 的增大而减小, \therefore 当 $x = 32$ 时, 所需成本最少. 此时, 成本 $= 20y = 20 \times (-10 \times 32 + 500) = 3\,600$ (元). 答: 每月的成本最少需要 $3\,600$ 元.

23. 解: (1) $\because y_1 = -2x^2 + 4x + 2 = -2(x - 1)^2 + 4$, \therefore 抛物线 C_1 的顶点坐标为 $(1, 4)$. \because 抛物线 C_1 与 C_2 顶点相同, $\therefore \frac{-m}{-1 \times 2} = 1, -1 + m + n = 4$. 解得 $m = 2, n = 3$. \therefore 抛物线 C_2 的表达式为 $y_2 = -x^2 + 2x + 3$; (2) 如图 1 所示, 设点 A 的坐标为 $(a, -a^2 + 2a + 3)$. $\therefore AQ = -a^2 + 2a + 3, OQ = a$, $\therefore AQ + OQ = -a^2 + 2a + 3 + a = -a^2 + 3a + 3 = -\left(a - \frac{3}{2}\right)^2 + \frac{21}{4}$. \therefore 当 $a = \frac{3}{2}$ 时, $AQ + OQ$ 有最大值, 最大值为 $\frac{21}{4}$;

(3) 如图 2 所示, 连结 BC , 过点 B' 作 $B'D \perp CM$, 垂足为 D . $\because B(-1, 4), C(1, 4)$, 抛物线的对称轴为 $x = 1$, $\therefore BC \perp CM, BC = 2$. $\therefore \angle BMB' = 90^\circ, \therefore \angle BMC + \angle B'MD = 90^\circ$. $\because B'D \perp MC, \therefore \angle MB'D + \angle B'MD = 90^\circ. \therefore \angle MB'D = \angle BMC$. 在 $\triangle BCM$ 和 $\triangle MDB'$ 中, $\begin{cases} \angle BMC = \angle MB'D, \\ \angle BCM = \angle MDB', \\ BM = MB', \end{cases} \therefore \triangle BCM \cong \triangle MDB'. \therefore BC = MD, CM = B'D$. 设点 M 的坐标为 $(1, a)$. 则 $B'D = CM = 4 - a, MD = CB = 2. \therefore$ 点 B' 的坐标为 $(a - 3, a - 2)$. $\therefore -(a - 3)^2 + 2(a - 3) + 3 = a - 2$, 整理得 $a^2 - 7a - 10 = 0$, 解得 $a = 2$ 或 $a = 5$. 当 $a = 2$ 时, M 的坐标为 $(1, 2)$, 当 $a = 5$ 时, M 的坐标为 $(1, 5)$. 综上所述, 当点 M 的坐标为 $(1, 2)$ 或 $(1, 5)$ 时, B' 恰好落在抛物线 C_2 上.



(第 23 题)

数学模拟试卷(八)

一、1. B 2. D 3. A 4. C 5. D 6. C 7. A 8. C 9. B 10. C

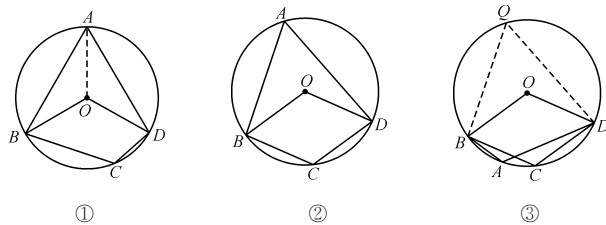
二、11. 12 12. 必然事件 13. $2\sqrt{21}$ 14. $\frac{\sqrt{5}-1}{2}$ 或 $\frac{3-\sqrt{5}}{2}$ 15. $r = 4.8$ 或 $6 < r \leq 8$ 16. ②③

三、17. 解: 落在圆内概率大. 理由: 圆的面积 $S_1 = \pi \times 1^2 = \pi$, 正六边形的面积 $S_2 = 6 \times \frac{\sqrt{3}}{4} \times 1^2 = \frac{3\sqrt{3}}{2}$, $\therefore S_1 > S_2$, \therefore 落在圆内概率大.

18. 解: (1) 把 $A(2, 0), B(0, -6)$ 代入, 得 $\begin{cases} 0 = -2 + 2b + c, \\ -6 = c, \end{cases}$ 解得 $\begin{cases} b = 4, \\ c = -6, \end{cases} \therefore$ 二次函数的表达式为 $y = -\frac{1}{2}x^2 + 4x - 6$; (2) 由(1)可知抛物线的对称轴为 $x = -\frac{b}{2a} = 4$, \therefore 点 C 的坐标为 $(4, 0)$, $\therefore S_{\triangle ABC} = \frac{1}{2}AC \cdot OB = \frac{1}{2} \times 2 \times 6 = 6$.

19. 解: (1) 证明: $\because AD$ 是 BC 上的高, $\therefore \angle ADB = \angle ADC = 90^\circ, \therefore \tan B = \cos \angle DAC, \therefore \frac{AD}{BD} = \frac{AD}{AC}, \therefore AC = BD$; (2) $\because \sin C = \frac{12}{13}, AD \perp BC, \therefore$ 不妨设 $AD = 12x$, 则 $AC = 13x, CD = 5x$, 由(1)可知 $BD = AC = 13x, \therefore BC = 12 + 13x + 5x = 12$, 解得 $x = \frac{2}{3}, \therefore AD = 12x = 12 \times \frac{2}{3} = 8$.

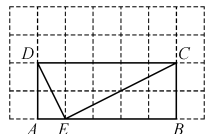
20. 解: (1) 如图①, 连结 AO . $\because \angle BOD = 120^\circ, \therefore \angle BAD = 60^\circ, \therefore OA = OB = OD, \therefore \angle OBA = \angle OAB, \angle ODA = \angle OAD, \therefore \angle OBA + \angle ODA = \angle OAB + \angle OAD = \angle BAD = 60^\circ$;



第 20 题

(2) 当点 A 在优弧 \widehat{BAD} 上, 如图②, 连结 OC , \because 四边形 $OBCD$ 为平行四边形, $\therefore BC=OD$, $\therefore BC=OD=OB=OC$, $\therefore \triangle OBC$ 为正三角形, 易得 $\angle BOD=120^\circ$, 由(1)可知 $\angle OBA+\angle ODA=60^\circ$. 当点 A 在劣弧 \widehat{BCD} 上, 如图③所示, 由上可知 $\angle BCD=\angle BOD=120^\circ$, $\therefore \angle BAD=120^\circ$, $\therefore \angle OBA+\angle ODA=120^\circ$.

21. 解: 方法一: (1) 如图, 设 $AE=x$, 则 $BE=5-x$, $\because \triangle ADE \sim \triangle BEC$, $\therefore \frac{2}{5-x} = \frac{x}{2}$, 解得 $x_1=1, x_2=4$, $\therefore AE=1$ 或 $AE=4$. 当 $AE=1$ 或 $AE=4$ 时, 此时 $\triangle ADE \sim \triangle ECD \sim \triangle BEC$; (2) 假设存在点 E, 由(1)可知 $\frac{3}{5-x} = \frac{x}{3}$, 即 $x^2-5x+9=0$, $\therefore b^2-4ac=25-36=-11<0$, \therefore 原方程无实数解, 即不存在这样的点 E. 方法二: 也可用以 CD 为直径的圆与线段 AB 有无交点进行说明.

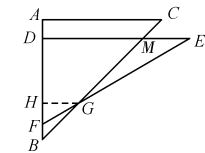


(第 21 题)

22. 解: (1) 当二次函数 $y=kx^2-4kx+3(k \neq 0)$ 与 x 轴只有一个公共点时 $(-4k)^2-4k \times 3=0$, 解得: $k=0$ 或 $k=\frac{3}{4}$, $\therefore k \neq 0$, $\therefore k=\frac{3}{4}$; (2) 结论①: 当 $x=0$ 时, $y=3$, 所以与 y 轴交点 $(0,3)$ 不变, 正确; 结论②: 对称轴 $x=-\frac{b}{2a}=-\frac{-4k}{2k}=2$ 不变, 正确; 结论③: 开口向上即 $k>0$, 顶点纵坐标 $y=-4k+3$ 无法确定, 错误; 结论④: $y=kx(x-4)+3$, 当 $x=0, y=3$, 当 $x=4, y=3$, \therefore 抛物线过定点 $(0,3), (4,3)$, 正确.

23. 解: (1) 证明: 当 EF 经过点 C 时. 在 $\text{Rt}\triangle DEF$ 中, $\tan \angle DFE = \frac{DE}{DF} = \sqrt{3}$, $\therefore \angle DFE = 60^\circ$.

在 $\text{Rt}\triangle AFC$ 中, $AC=b, \angle AFC=60^\circ$, $\therefore CF = \frac{2\sqrt{3}}{3}b$, 则 $DE=FC$; (2) 如图, 当 $0 \leq x \leq \frac{b}{3}$ 时, 过 G 作 $GH \perp BA$ 交 BA 于点 H, 由(1)可知 $\angle GFH=60^\circ$, $\tan 60^\circ = \frac{HG}{HF}$, 即 $HF = \frac{\sqrt{3}}{3}HG$, 在 $\triangle BHG$ 中, $\because \angle B=45^\circ$, $\therefore HG=HB$, 则 $HG = \frac{3+\sqrt{3}}{2}x$, 又 $\because DM=DB = \frac{2}{3}b+x$



(第 23 题)

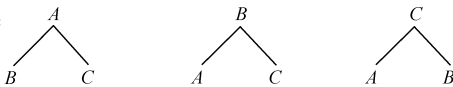
且 $AD = \frac{b}{3} - x$, $S_{\text{四边形DFGM}} = S_{\triangle ABC} - S_{\triangle BFG} - S_{\text{四边形ADMC}} = \frac{1}{2}b^2 - \frac{1}{2}x \cdot \frac{3+\sqrt{3}}{2}x - \frac{1}{2}(\frac{2}{3}b+x+b)(\frac{b}{3}-x) = -\frac{\sqrt{3}+1}{4}x^2 + \frac{2}{3}bx + \frac{2}{9}b^2$; 同理: 当 $\frac{b}{3} < x < \frac{3-\sqrt{3}}{3}b$, $y = -\frac{3+\sqrt{3}}{4}x^2 + \frac{b^2}{2}$, 当 $\frac{3-\sqrt{3}}{3}b \leq x \leq b$, $y = \frac{\sqrt{3}(b-x)^2}{2}$.

数学模拟试卷(九)

一、1. C 2. C 3. A 4. B 5. B 6. D 7. C 8. D 9. A 10. A

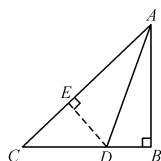
二、11. $\frac{4}{3}$ 12. $\frac{1}{10}$ 13. $50^\circ \leq \angle BPD \leq 100^\circ$ 14. 10π 15. -6 或 0 16. (1) $\frac{1}{2}$ (2) $\frac{2\sqrt{34}}{17}$

三、17. 解: (1) 当 $x=0$ 时, $y_1=x-2=-2, y_2=x^2-x-2=-2$, 则 A 点在直线和抛物线上; 当 $x=2$ 时, $y_1=x-2=0, y_2=x^2-x-2=0$, 则 B 点在直线和抛物线上; 当 $x=-1$ 时, $y_1=x-2=-3, y_2=x^2-x-2=0$, 则 C 点在直线上, 不在抛物线, 所以在 A, B, C 三个点中任取一个点, 这个点既在直线 $y_1=x-2$ 上又在抛物线 $y_2=x^2-x-2$ 上的概率为 $\frac{2}{3}$; (2) 画树状图:



共有 6 种等可能的结果, 其中两点都落在抛物线 $y_2=x^2-x-2$ 上的结果数为 2, 所以两点都落在抛物线 $y_2=x^2-x-2$ 上的概率为 $\frac{2}{6} = \frac{1}{3}$.

18. 解:(1)如图,过点 D 作 $DE \perp AC$ 于点 E . $\because CD=8$ m, $\angle C=45^\circ$, $\therefore CE=DE=\frac{CD}{\sqrt{2}}=4\sqrt{2}$ m;
 (2) $\because \angle C=45^\circ, \angle ADB=75^\circ, \therefore \angle CAD=30^\circ, \therefore DE=4\sqrt{2}$ m, $\therefore AE=4\sqrt{6}$ m, $\therefore AC=(4\sqrt{2}+4\sqrt{6})$ m. $\because \angle C=45^\circ, \angle B=90^\circ, \therefore AB=\frac{CA}{\sqrt{2}}=\frac{4\sqrt{2}+4\sqrt{6}}{\sqrt{2}}=(4+4\sqrt{3})$ m.

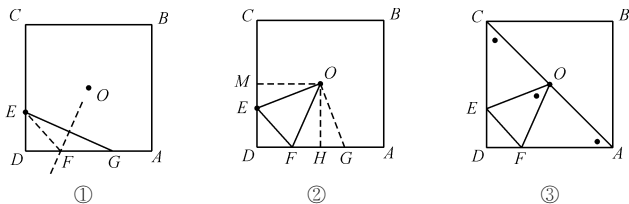


19. 解:证明:(1) \because 四边形 $ABCD$ 是平行四边形, $\therefore \angle B=\angle D, \therefore \angle ECA=\angle D, \therefore \angle ECA=\angle B, \therefore \angle E=\angle E, \therefore \triangle EAC \sim \triangle ECB$; (2) \because 四边形 $ABCD$ 是平行四边形, $\therefore CD \parallel AB$ 即 $CD \parallel AE, \therefore \frac{CD}{AE}=\frac{DF}{AF}, \therefore DF=AF, \therefore CD=AE, \therefore$ 四边形 $ABCD$ 是平行四边形, $\therefore AB=CD, \therefore AE=AB, \therefore BE=2AE, \therefore \triangle EAC \sim \triangle ECB, \therefore \frac{AE}{CE}=\frac{CE}{BE}=\frac{AC}{BC}, \therefore CE^2=AE \cdot BE=\frac{1}{2}BE^2$, 即 $\frac{CE}{BE}=\frac{\sqrt{2}}{2}, \therefore \frac{AC}{BC}=\frac{\sqrt{2}}{2}$.

20. 解:(1) \because 在正方形纸上剪去 4 个全等的直角三角形, $\therefore \angle AHE=\angle DGH, \angle DGH+\angle DHG=90^\circ, HG=HE, \therefore \angle EHG=180^\circ-\angle AHE-\angle DHG, \therefore \angle EHG=90^\circ$, 四边形 $EFGH$ 为正方形, 在 $\triangle AEH$ 中, $AE=x, AH=BE=AB-AE=2-x, \angle A=90^\circ, \therefore HE^2=AE^2+AH^2=x^2+(2-x)^2=2x^2-4x+4$, 正方形 $EFGH$ 的面积 $y=HE^2=2x^2-4x+4, \therefore AE, AH$ 为正, $\therefore 0 < x < 2$; (2)将 $y=3$ 代入 $y=2x^2-4x+4$ 中, 整理得 $2x^2-4x+1=0$, 解得: $x_1=1+\frac{\sqrt{2}}{2}, x_2=1-\frac{\sqrt{2}}{2}$, 故四边形 $EFGH$ 的面积为 3 cm^2 时的 x 的值为 $1+\frac{\sqrt{2}}{2}$ 或 $1-\frac{\sqrt{2}}{2}$; (3)四边形 $EFGH$ 的面积为 $y=2x^2-4x+4=2(x-1)^2+2(0 < x < 2)$, \therefore 当 $x=1$ 时, 四边形 $EFGH$ 的面积最小, 为 2 cm^2 .

21. 解:(1)证明:如图,连结 DE . $\because BD$ 是 $\odot O$ 的直径, $\therefore \angle DEB=90^\circ. \therefore E$ 是 AB 的中点, $\therefore DA=DB, \therefore \angle 1=\angle B, \therefore \angle B=\angle F, \therefore \angle 1=\angle F$; (2) $\because \angle 1=\angle F, \therefore AE=EF=2\sqrt{5}, \therefore AB=2AE=4\sqrt{5}$, 在 $\text{Rt}\triangle ABC$ 中, $AC=AB \cdot \sin B=4, \therefore BC=8$, 设 $CD=x$, 则 $AD=BD=8-x, \therefore AC^2+CD^2=AD^2$, 即 $4^2+x^2=(8-x)^2, \therefore x=3$, 即 $CD=3$.

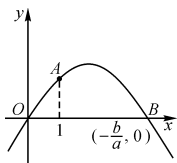
22. 解:(1)3; (2)①如图①所示:



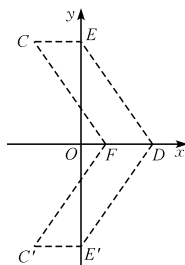
(第 22 题)

$DG=CE$ 或 $DE=GA$; ②如图②, $OG=OE, DE=AG, \therefore \triangle EDF$ 的周长等于 AD 的长, $\therefore EF=FG, \therefore \triangle EOF \cong \triangle GOF$ (SSS), $\therefore \angle EOF=\angle FOG, \therefore \triangle OEM \cong \triangle OHG$ (HL), $\therefore \angle EOG=90^\circ, \therefore \angle EOF=45^\circ$; ③如图③, 设 $AF=8k, CE=9k, \therefore \angle EOF=\angle ECO=\angle FAO=45^\circ, \therefore \angle FOA=\angle CEO, \therefore \triangle AFO \sim \triangle COE, \therefore \frac{OF}{OE}=\frac{AF}{CO}=\frac{AO}{CE}, \therefore OC^2=CE \cdot AF, \therefore OC=6\sqrt{2}k, \therefore \frac{OF}{OE}=\frac{2\sqrt{2}}{3}$.

23. 解:(1)(3,0);
 (2)如图①所示;
 (3)①如图②,



①



②

(第 23 题)

\because 特征点 C 为直线 $y=-4x$ 上一点, $\therefore b=-4a, \therefore$ 抛物线 $y=ax^2+bx$ 的对称轴与 x 轴交于点 D, \therefore 对称轴 $x=-\frac{b}{2a}=2$, 点 D 的坐标为 $(2,0), \therefore$ 点 F 的坐标为 $(1,0), \therefore DF=1. \therefore$ 特征直线 $y=ax+b$ 交 y 轴于点 E, \therefore 点 E 的坐标为 $(0,b), \therefore$ 点 C 的坐标为 $(a,b), \therefore CE \parallel DF, \therefore DE \parallel CF, \therefore$ 四边形 $DECF$ 为平行四边形, $\therefore CE=DF$

$=1, \therefore a = -1, \therefore$ 特征点 C 的坐标为 $(-1, 4)$; ② $\because \tan \angle ODE = \frac{OE}{OD} = \left| \frac{b}{-2a} \right| = |2a|, \therefore \frac{1}{4} < |a| < 1, \therefore$
 $\frac{1}{4} < a < 1$ 或 $-1 < a < -\frac{1}{4}, \therefore DE \parallel CF, \therefore -\frac{b}{2a} - 1 = -a, \text{即 } b = 2a^2 - 2a, \therefore -\frac{1}{2} \leq b < 0$ 或 $\frac{5}{8} < b < 4$.

数学模拟试卷(十)

一、1. D 2. D 3. B 4. B 5. A 6. C 7. A 8. B 9. C 10. A

二、11. -3 12. $5 \tan 55^\circ$ 米 13. $x = 3$ $0 \leq x < 3$ 14. 2 15. $\frac{100\pi}{3} \text{cm}^2$ 16. $2 + 2\sqrt{5}$ 或 $2 - 2\sqrt{5}$

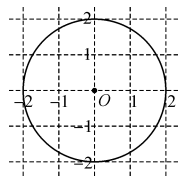
三、17. 解:(1)作图略;(2) $EG \perp DF$ 由已知可得: FD 为 $\angle ADG$ 的角平分线, 且 $AD \parallel BG, \therefore \angle EFG = \angle EDG, \therefore$
 $FG = DG, \text{又 } \because E$ 为 AB 中点, $\therefore AE = BE, \angle AED = \angle FEB, \angle FEB = \angle EAD, \therefore \triangle EBF \cong \triangle EAD, E$ 为 DF
 中点, 且 $\triangle FDG$ 为等腰三角形, $\therefore EG \perp DF$.

18. 解:(1)当 $2m - 6 = m - 2$ 时, $m = 4$, 此时 $2m - 6 = 2$, 此数为 4; (2)当 $2m - 6 = -(m - 2)$ 时, $m = \frac{8}{3}$, 此时 $2m - 6 = 2 \times \frac{8}{3} - 6 = -\frac{2}{3} < 0$, 不合题意舍去. 由(1)(2)得, 此数只能为 4.

19. 解:(1)能得到 25 个数组; (2)点 P 落在圆内的概率是 $\frac{9}{25}$.

20. 解:待定系数法得直线 AB 的解析式为 $y = -x - 1$, 平移后 $y = -x - 1 + m$, 联立解得

$$\begin{cases} y = -x - 1 + m, \\ y = 2x - 4, \end{cases} \text{得} \begin{cases} x = \frac{m+3}{3}, \\ y = \frac{2m-6}{3}, \end{cases} \text{因交点在第一象限, 所以} \begin{cases} \frac{m+3}{3} > 0, \\ \frac{2m-6}{3} > 0, \end{cases} \text{得 } m > 3; \text{(2)作} \quad \text{(第 19 题)}$$

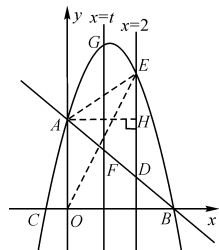


$AB \perp$ 直线 $y = 2x - 4$, 垂足为 B , 此时线段 AB 最短. 过点 B 作 $BE \perp x$ 轴, 垂足为 E , 易证 $\triangle ABE \sim \triangle DCO$, 即 $\frac{AE}{DO} = \frac{BE}{CO}$. 因为 $CO = 2, DO = 4, AE = 1 + x, BE = 4 - 2x$, 所以 $\frac{1+x}{4} = \frac{4-2x}{2}$, 解得 $x = \frac{7}{5}$, 所以 $B(\frac{7}{5}, -\frac{6}{5})$.

21. 解:(1) $\because AB = AC, AE$ 是 $\triangle ABC$ 中 BC 边上的高线, $\therefore BE = CE, AE \perp BC, \therefore DC = BD$. 又 $\because AD = AD, \therefore$
 $\triangle ADB \cong \triangle ADC$. (2) $\because \triangle AEB \sim \triangle BED, \therefore \angle BAE = \angle DBE, \therefore \cos \angle DBE = \frac{2}{3}, \therefore \cos \angle BAE = \frac{2}{3}$. 在 Rt
 $\triangle BAE$ 中, $\cos \angle BAE = \frac{2}{3} = \frac{AE}{AB}, \therefore 2AB = 3AE$. 又 $BC = 8, E$ 为中点, $\therefore BE = 4, \therefore AE^2 + BE^2 = AB^2, \therefore AE^2 +$
 $16 = \frac{9}{4} AE^2$, 解得 $AE = \frac{8\sqrt{5}}{5}$.

22. 解:(1)若 $m = 2$, ①则抛物线的解析式为 $y = -x^2 + \frac{7}{2}x + 2$, 得 $A(0, 2), B(4, 0), C$

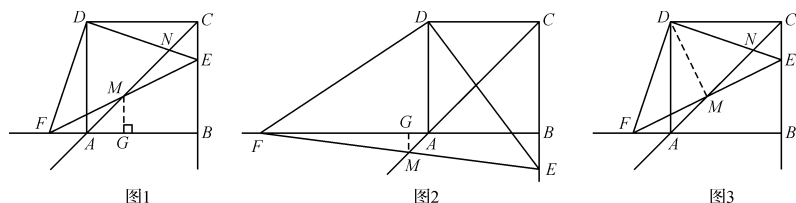
$(-\frac{1}{2}, 0)$ 所以直线 AB 的解析式为 $y = -\frac{1}{2}x + 2$. ②易得 $E(2, 5), D(2, 1), G(t, -t^2 + \frac{7}{2}t + 2), F(t, -\frac{1}{2}t + 2)$, 所以 $DE = 4, FG = -t^2 + 4t$, 因 $FG : DE = 3 : 4$, 所以 $-t^2 + 4t = 3$, 解得 $t_1 = 1, t_2 = 3$. (2)抛物线的解析式为 $y = -x^2 + \frac{7}{2}x + m$, 易得 $A(0,$



(第 22 题)

$m), E(2, m+3)$, 过点 A 作 $AH \perp DE$ 于点 H , 可得 $H(2, m)$. 因 EO 平分 $\angle AED$, 所以 $\angle AEO = \angle DEO$, 又因为 $DE \parallel AO$, 所以 $\angle DEO = \angle AOE$, 即 $\angle AEO = \angle AOE$, 所以 $AO = AE$. 在直角 $\triangle AHE$ 中, $AE^2 = AH^2 + EH^2 = 2^2 + 3^2 = 13$, 即 $|m| = AO = AE = \sqrt{13}, \therefore m = \pm \sqrt{13}$.

23. 解:(1)由正方形得 $AD = CD, \angle DAF = \angle DCE = 90^\circ$, 由速度相同得 $AF = CE$, 所以 $\triangle ADF \cong \triangle CDE$, 得 $\angle FDA = \angle CDE$, 所以 $\angle FDA + \angle ADE = \angle CDE + \angle ADE = \angle ADC = 90^\circ$, 所以 $\angle FDE = 90^\circ$, 即 $DE \perp DF$. (2)当 $0 < x < 4$ 时, 如图 1, 过点 M 作 $MG \perp AB$, 由 $CB \perp AB$ 得 $\triangle FMG \sim \triangle FEB$, 得 $\frac{FG}{FB} = \frac{MG}{BE}$, 因为 $MG = AG$, 设 $MG = h$, 所以 $FG = FA + AG = FA + MG = x + h, FB = x + 4, BE = 4 - x$, 得 $\frac{x+h}{x+4} = \frac{h}{4-x}$, 得 $h = \frac{4-x}{2}, y = \frac{1}{2}x \cdot \frac{4-x}{2} = -\frac{1}{4}x^2 + x$. 当 $x > 4$ 时, 如图 2, 过点 M 作 $MG \perp AB$, 同理可得 $\frac{FG}{FB} = \frac{MG}{BE}$, 因为 $MG = AG$, 设 $MG = h$, 所以 $FG = FA - AG = FA - MG = x - h, FB = x + 4, BE = x - 4$, 得 $\frac{x-h}{x+4} = \frac{h}{x-4}$, 得 $h = \frac{x-4}{2}, y = \frac{1}{2}x \cdot \frac{x-4}{2} = \frac{1}{4}x^2 - x$.



(第 23 题)

(3)由(2)得 $\triangle FMG \sim \triangle FBE$, $\frac{FG}{FB} = \frac{MG}{BE} = \frac{1}{2}$, 所以 $\frac{FM}{FE} = \frac{1}{2}$, 即 M 为 FE 中点, 又由 $\triangle ADF \cong \triangle CDE$ 得 $DF = DE$, 连结 DM, DM 为 $\angle FDE$ 平分线, 即 $\angle MDE = 45^\circ$. 又 $\angle DCM = \angle DAN = 45^\circ$, 所以 $\angle MDC = \angle MND$, 所以 $\triangle NAD \sim \triangle DCM$, 得 $\frac{NA}{CD} = \frac{AD}{MC}$, 即 $NA \cdot MC = AD \cdot CD = 16$.

数学模拟试卷(十一)

一、1. D 2. B 3. C 4. C 5. D 6. D 7. C 8. A 9. A 10. A

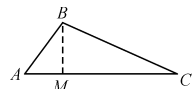
二、11. $\frac{\sqrt{5}}{2}$ 12. $x \geq 3$ 或 $x < 2$ 13. $\frac{2}{3}$ 14. ①③④ 15. $\frac{\sqrt{3}}{2}$ 16. 4 或 5 3 或 4 或 5 或 6

三、17. 解: 由二元一次方程组解得 $x = \frac{2}{3}, y = \frac{4}{5}$ 原式 $= \frac{xy}{x^2 - y^2} \cdot \frac{x+y}{xy} + 1 = \frac{1}{x-y} + 1$, 代入得原式 $= -\frac{13}{2}$.

18. 解: 连结 CA, DA, $\because AB = AE, \angle B = \angle E, BC = ED, \therefore \triangle CBA \cong \triangle DEA, \therefore CA = DA$. 又 $\because AF \perp CD, \therefore CF = DF$.

19. 解: 过 B 作 $BM \perp AC$ 于 M, 设 $MC = x, 1 - (\sqrt{5} - x)^2 = (\sqrt{2})^2 - x^2$, 解得 $x = \frac{3\sqrt{5}}{5}$, 则 $BM =$

$$\frac{\sqrt{5}}{5}, \sin \angle ACB = \frac{\sqrt{5}}{5\sqrt{2}} = \frac{\sqrt{10}}{10}.$$



(第 19 题)

20. 解: (1) $\Delta = 16 - 12 = 4 > 0$, 有两个不相等的实数根; (2)①配方法: $(x-2)^2 - 1 = 0, x_1 = 3, x_2 = 1$; ②因式分解法: $(x-1)(x-3) = 0, x_1 = 1, x_2 = 3$; (3)一个一元二次方程, 两个一元一次方程; (4) $x(x-1)(x+1) = 0, x_1 = 0, x_2 = 1, x_3 = -1$.

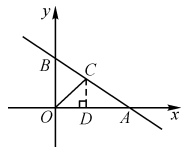
21. 解: (1) $\because B(0, 3), \therefore OB = 3$. 又 $\because \angle AOB = 90^\circ, \cos \angle BAO = \frac{4}{5}, \therefore$ 可设 $AB = 5n, OA = 4n, \therefore OA^2 + OB^2 =$

AB^2 , 即 $16n^2 + 9 = 25n^2, \therefore n = 1, \therefore OA = 4, A(4, 0)$, 把 $A(4, 0)$ 和 $B(0, 3)$ 代入, 得 $\begin{cases} 4k + b = 0, \\ b = 3, \end{cases}$ 解得

$$\begin{cases} k = -\frac{3}{4}, \\ b = 3, \end{cases} \therefore \text{一次函数的解析式为 } y = -\frac{3}{4}x + 3; \text{ (2)过点 } C \text{ 作 } CD \perp OA, \text{ 设 } OD = a.$$

$\because OC$ 是 $\angle AOB$ 的平分线, $\angle AOB = 90^\circ, \therefore \angle COD = \frac{1}{2} \angle AOB = 45^\circ$. 又 $\because CD \perp OA, \therefore$

$\triangle CDO$ 是等腰直角三角形, $\therefore CD = OD = a, \therefore C(a, a)$. 又 \because 点 C 在直线 AB 上, $\therefore -\frac{3}{4}a$



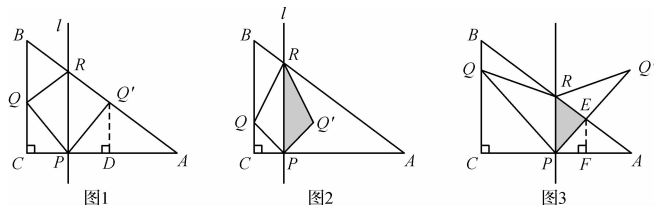
(第 20 题)

$$+ 3 = a, \therefore a = \frac{12}{7}, \therefore C\left(\frac{12}{7}, \frac{12}{7}\right), \text{ 把 } C\left(\frac{12}{7}, \frac{12}{7}\right) \text{ 代入 } y = \frac{m}{x}, \text{ 得 } m = \frac{144}{49}.$$

22. 解: (1)当 $m = 2, n \neq -1$ 时, 是二次函数; 当 $m = 1, n \neq -2$ 时或者当 $m \neq 0, n = -1$ 时, 是一次函数; 当 $m = 1, n = 1$ 时, 是正比例函数; 不可能是反比例函数. 一定与 x 轴有交点理由如下: 若是一次函数, 直线必定与 x 轴有交点; 若是二次函数, 判别式 $\Delta = 2^2 - 4(n+1)(1-n) = 4n^2 \geq 0$, 与 x 轴有交点; (2)①若是二次函数, 则 $m = 2, n \neq -1, y = (n+1)x^2 + 2x + 1 - n, n > -1$, 开口向上, 对称轴为直线 $x = \frac{-1}{n+1}$ 在 y 轴左侧, 当 $\frac{-1}{n+1} \leq x < 0$ 时, y 随 x 增大而增大, 与命题不符, 故原命题为假命题; ②由 $y = (n+1)x^2 + 2x + 1 - n = (x^2 - 1)n + x^2 + 2x + 1$ 可得, 过定点 $(-1, 0)$ 和 $(1, 4)$ (也可用赋值法, n 取任意值解方程组).

23. 解: (1) $b = 2$; 形状是等腰三角形; (2)①过点 Q' 作 $Q'D \perp AC$ (如图 1). $\because \triangle PQR$ 与 $\triangle P'Q'R$ 关于直线 l 对称 又 $\because a = b, \therefore PC = PD = a, \therefore AD = 8 - 2a, \therefore \tan A = \frac{Q'D}{AD} = \frac{BC}{AC}$ 即, $\frac{a}{8-2a} = \frac{6}{8}$, 解得 $a = \frac{12}{5}$.

②(I) 当 $0 \leq a \leq \frac{12}{5}$ 时, 重叠部分为 $\triangle PQ'R$ (如图 2). $\therefore \tan A = \frac{RP}{AP} = \frac{BC}{AC}, \therefore \frac{RP}{8-a} = \frac{6}{8}$, 即 $RP = \frac{3}{4}(8-a), \therefore$



(第 23 题)

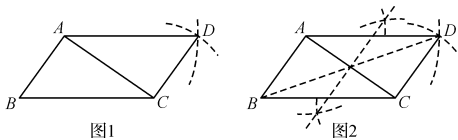
$S = \frac{1}{2} \cdot \frac{3}{4} (8-a) \cdot a$, 即 $S = -\frac{3}{8}a^2 + 3a$, ($0 \leq a \leq \frac{12}{5}$); (II) 当 $\frac{12}{5} < a \leq 6$ 时, 重叠部分为 $\triangle PER$ (如图 3), $\because \angle C = 90^\circ, a = b, \therefore \angle QPC = 45^\circ, \therefore \angle Q'PA = 45^\circ, \therefore PF = EF$, 不妨设 $EF = m$, 则 $PF = m, AF = \frac{4}{3}m$. 又 $\because CP + PF + AF = 8, \therefore a + m + \frac{4}{3}m = 8, \therefore m = \frac{3}{7}(8-a), \therefore S = \frac{1}{2} \times \frac{3}{4} (8-a) \cdot \frac{3}{7} (8-a)$, 即 $S = \frac{9}{56} (8-a)^2$ ($\frac{12}{5} < a \leq 6$).

数学模拟试卷(十二)

一、1. C 2. D 3. C 4. C 5. B 6. B 7. D 8. B 9. C 10. D

二、11. $\sqrt{3} + 3\sqrt{2}$ 12. $\frac{29}{3}$ 13. $\frac{1}{12}$ 14. $\frac{1}{2}$ 15. 0 16. ± 1 或 $\pm \frac{\sqrt{3}}{3}$

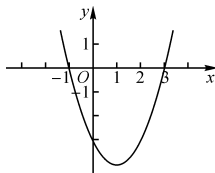
三、17. 解: 方法 1 如图 1, 依据是两组对边分别相等的四边形是平行四边形;



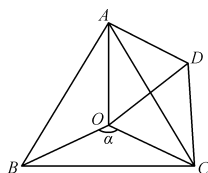
方法 2 如图 2, 依据是对角线互相平分的四边形是平行四边形.

18. 解: (1) 50 人. (2) 图略 (3) 设八年级同学至少有 x 人, 则 $42 + 91\%(x-50) \geq 90\%x$, 解得 $x \geq 350$. 答: 八年级同学至少有 350 人.

19. 解: (1) 设 $y = a(x-1)^2 + k$, 将 $A(-2, 5), C(0, -3)$ 代入得 $a = 1, k = -4$. 即 $y = x^2 - 2x - 3$, 顶点坐标为 $(1, -4)$; (2) $y = 0$, 即 $x^2 - 2x - 3 = 0$, 解得 $x_1 = -1, x_2 = 3$, 即函数的图象与 x 轴的交点坐标为 $(-1, 0), (3, 0)$; (3) 当 $y < 0$ 时, $-1 < x < 3$.



(第 19 题)



(第 21 题)

20. 解: (1) $S = S_1 + S_2 = 27\pi + 54\pi = 81\pi$; (2) $\because S_1 = \frac{n_1 \pi R_1^2}{360}, \therefore n_1 = \frac{360 S_1}{\pi R_1^2} = \frac{360 \times 27\pi}{\pi \times 9^2} = 120^\circ$, 连结 CC' , 过点 D 作

CC' 的垂线, 垂足为 E , 则由垂径定理可知 $CE = C'E, \therefore CC' = 2CE = 2CD \times \sin 60^\circ = 2 \times 9 \times \frac{\sqrt{3}}{2} = 9\sqrt{3}$. 如左图,

同理可得另一最短路线为 18. $\because 9\sqrt{3} < 9 \times \sqrt{4} = 9 \times 2 = 18, \therefore$ 蚂蚁爬过的最短路线长为 $9\sqrt{3}$.

21. 解: (1) $\triangle OCD$ 是等边三角形. $\because \triangle BCO \cong \triangle ACD, \therefore OC = CD$ 又 $\because \angle OCD = 60^\circ, \therefore \triangle OCD$ 是等边三角形. (2) $\triangle AOD$ 不可能是等边三角形. 若 $\triangle AOD$ 是等边三角形, 则 $\angle ADO = 60^\circ, \therefore \triangle OCD$ 是等边三角形, $\therefore \angle DOC = \angle CDO = 60^\circ$. 即 $\angle ADC = 120^\circ$. 又 $\because \angle AOB + \angle \alpha + \angle COD + \angle AOD = 360^\circ$ 且 $\angle AOB = 105^\circ, \therefore \angle BOC = 360^\circ - 105^\circ - 60^\circ - 60^\circ = 135^\circ$. 这与已知 $\angle BOC = \angle ADC$ 矛盾.

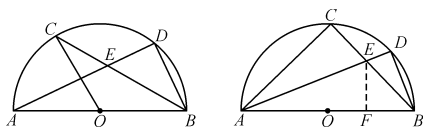
22. 解: (1) 如图 1, $OC \parallel DB, OB = OC, \therefore \angle DBC = \angle C = \angle CBA, \therefore \widehat{DC} = \widehat{AC}$, 又点 D 平分 $\widehat{BC}, \therefore \angle DBC = \angle C = \angle CBA = 30^\circ, \therefore \widehat{AC} = \frac{\pi}{3}, t = 2$. 在 $\text{Rt}\triangle ABD$ 中, $\angle D = 90^\circ, AB = 2, \therefore DB = 1, AD = \sqrt{3}$, 在 $\text{Rt}\triangle BDE$ 中, $\angle D = 90^\circ, BD = 1, \therefore DE = \frac{\sqrt{3}}{3}, \therefore AE = \frac{2\sqrt{3}}{3}, \frac{DB}{AE} = \frac{\sqrt{3}}{2}$; (2) 如图 2, 过点 E 作 $EF \perp AB$ 于点 F , 理由: 当 $t = 3$ 时, \widehat{AC}

$$= \frac{\pi}{2}, \angle ABC = 45^\circ, \therefore AC = BC = \sqrt{2}, BF = EF = CE = 2 - \sqrt{2}, EB$$

$$= \sqrt{2}BF = 2\sqrt{2} - 2, \therefore AE^2 = (\sqrt{2})^2 + (2 - \sqrt{2})^2 = 8 - 4\sqrt{2}. \text{ 由}$$

$$\triangle ACE \sim \triangle BDE \text{ 得: } \frac{DB}{AC} = \frac{BE}{AE}, \therefore DB = \frac{AC \cdot BE}{AE}, \therefore \frac{DB}{AE} =$$

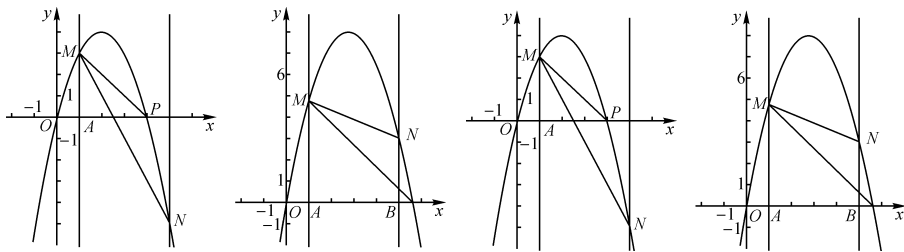
$$\frac{AC \cdot BE}{AE^2} = \frac{\sqrt{2} \cdot (2\sqrt{2} - 2)}{8 - 4\sqrt{2}} = \frac{1}{2}.$$



(第 22 题)

23. 解:(1)由题意得 $O(0,0), P(t,0)$, 代入 $y = -x^2 + bx + c$, 得 $c = 0, -t^2 + bt = 0$, 即 $b = t$. 即 $y = -x^2 + tx$. (2)

当 $t > 1$ 时, ① $M(1, t-1)$, 即 $AM = t-1, AP = t-1$, 即 $AM = AP, \angle PAM = 45^\circ, \sin \angle MPO = \sin 45^\circ = \frac{\sqrt{2}}{2}$ 是定值.



(第 23 题)

②当 $1 < x \leq 5$ 时, $N(5, 5t-25)$, 如图 1, 过点 N 作 AM 的垂线, 垂足为 B , $S_{\triangle MPN} = S_{\triangle APM} + S_{\text{梯形} ABNP} - S_{\triangle MBN}, = \frac{1}{2}(t$

$-1)^2 + \frac{1}{2}(t-1+4) \times (25-5t) - \frac{1}{2}(t-1-5t+25) \times 4 = -2t^2 + 12t - 10$, 当 $t > 5$ 时, 如图 2, $S_{\triangle MPN} = S_{\text{梯形} MABN} +$

$S_{\triangle MBP} - S_{\triangle APM} = \frac{1}{2}(t-1+5t-25) \times 4 + \frac{1}{2}(5t-25)(t-5) - \frac{1}{2}(t-1)^2 = 2t^2 - 12t + 10$; ③存在这样的 t 值, 使

得以 O, M, N, P 为顶点的四边形为梯形. 1°当 $MP \parallel ON$ 时, 如图 3, $\because \angle PAM = 45^\circ, \therefore \angle PAN = 45^\circ$, 即 $N(5, -5)$, 代入 $y = -x^2 + tx$ 得 $-25 + 5t = -5$. 解得 $t = 4$; 2°当 $MN \parallel OP$ 时, 如图 4, 则 M, N 关于对称轴 $x = 3$ 对称, 即 $-\frac{t}{2 \times (-1)} = 3$, 得 $t = 6$. 综上, 当 $t = 4$ 或 $t = 6$ 时, 以 O, M, N, P 为顶点的四边形为梯形.

数学模拟试卷(十三)

一、1. A 2. C 3. C 4. A 5. B 6. B 7. C 8. A 9. C 10. B

二、11. $-\sqrt{6}$ 12. $2\sqrt{2}$ 13. 6 14. $(0, 2), (1, 0)$ 15. $\frac{15}{4}$ 16. $\frac{\sqrt{6}}{3}$

三、17. 解: 容易求得 $AB = 3\sqrt{5}$, 因为 $\triangle AEF \sim \triangle BED$, 且 $\frac{AF}{BD} = \frac{5}{4}$. 所以 $\frac{AE}{BE} = \frac{AF}{BD} = \frac{5}{4}$,

$$\text{即 } \frac{AE}{AB} = \frac{5}{9}, \text{ 所以 } AE = \frac{5}{9}AB = \frac{5}{9} \times 3\sqrt{5} = \frac{5\sqrt{5}}{3}.$$

18. 解:(1)

$x =$	-2	-1	1	2
$y =$	$3a-3$	-1	3	$3a+5$

(2)发现: 函数 $y = ax^2 + 2x + (1-a)$ 的图像经过两个定点 $(-1, -1), (1, 3)$. 证明: 令 $a = 0$, 则 $y = 2x + 1; a = 1$,

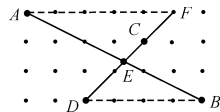
则 $y = x^2 + 2x$. 联立解得 $x = 1$ 或 -1 . 所以 $\begin{cases} x=1 \\ y=3 \end{cases}$ 或 $\begin{cases} x=-1 \\ y=-1 \end{cases}$, 经过检验满足函数解析式 $y = ax^2 + 2x + (1-a)$.

所以函数经过定点 $(-1, -1), (1, 3)$.

19. 解:(1) $(a, b) = (2, 3) = (2, 4) = (3, 4) = (4, 3)$; (2) 共有 8 个交点: $(-1, 0); (-\frac{1}{2}, \frac{1}{2}); (-\frac{1}{3}, \frac{2}{3}); (0,$

$2); (\frac{1}{3}, \frac{10}{3}); (1, 4); (1, 5); (2, 6)$. 满足条件的仅 $(2, 6)$ 一个点, 故 $P = \frac{1}{8}$.

20. 证明: 把已知代数式整理成关于 x 的二次三项式, 得原式 $= 3x^2 + 2(a+b+c)x + ab + ac + bc$. \because 它是完全平方



(第 17 题)

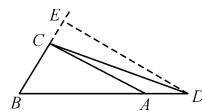
式, $\therefore \Delta=0$. 即 $4(a+b+c)^2 - 12(ab+ac+bc) = 0$. $\therefore 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 0, (a-b)^2 + (b-c)^2$

$+ (c-a)^2 = 0$. 要使等式成立, 必须且只需 $\begin{cases} a-b=0, \\ b-c=0, \\ c-a=0, \end{cases}$ 解这个方程组, 得 $a=b=c$.

21. 解: (1) 作 $DE \perp BC$, 垂足为 E . 由勾股定理得 $CD^2 - BD^2 = (CE^2 + DE^2) - (BE^2 + DE^2) = CE^2 - BE^2 = (CE - BE)BC$. 所以 $\frac{CD^2 - BD^2}{BC^2} = \frac{CE - BE}{BC} = \frac{CE}{BC} - \frac{BE}{BC}$. 因为 $DE \parallel AC$, 所以 $\frac{CE}{BC} = \frac{AD}{AB}, \frac{BE}{BC} = \frac{BD}{AB}$. 故 $\frac{CD^2 - BD^2}{BC^2} = \frac{AD}{AB} - \frac{BD}{AB} = \frac{AD - BD}{AB}$.

(2) 当点 D 与点 A 重合时, 第(1)小题中的等式仍然成立. 此时有 $AD=0, CD=AC, BD=AB$. 所以 $\frac{CD^2 - BD^2}{BC^2} = \frac{AC^2 - AB^2}{BC^2} = \frac{-BC^2}{BC^2} = -1, \frac{AD - BD}{AB} = \frac{-AB}{AB} = -1$. 从而第(1)小题中的等式成立.

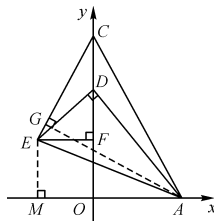
(3) 当点 D 在 BA 的延长线上时, 第(1)小题中的等式不成立. 作 $DE \perp BC$, 交 BC 的延长线于点 E , 则 $\frac{CD^2 - BD^2}{BC^2} = \frac{CE^2 - BE^2}{BC^2} = -\frac{CE + BE}{BC} = -1 - \frac{2CE}{BC}$, 而 $\frac{AD - BD}{AB} = \frac{-AB}{AB} = -1$, 所以 $\frac{CD^2 - BD^2}{BC^2} \neq \frac{AD - BD}{AB}$.



(第 21 题)

22. 解: (1) $\because \angle EFD = \angle EDA = 90^\circ, \therefore \angle DEF + \angle EDF = 90^\circ, \angle EDF + \angle ODA = 90^\circ. \therefore \angle DEF = \angle ODA. \therefore \triangle EDF \sim \triangle DAO. \therefore \frac{EF}{DO} = \frac{ED}{DA}. \therefore \frac{EF}{DO} = \frac{1}{2}. \therefore OD = t, \therefore \frac{EF}{t} = \frac{1}{2}, \therefore EF = \frac{1}{2}t$. 同理 $\frac{DF}{OA} = \frac{ED}{DA}, \therefore DF = 2, \therefore OF = t - 2$.

(2) $\because OC = 8$. 如图, 连结 EC, AC , 过 A 作 EC 的垂线交 CE 于 G 点. $\because \angle ECA = \angle OAC, \therefore \angle OAC = \angle GCA$. 在 $\triangle CAG$ 与 $\triangle OCA$ 中, $\because \angle OAC = \angle GCA, AC = CA, \angle ECA = \angle OAC, \therefore \triangle CAG \cong \triangle OCA. \therefore CG = AO = 4, AG = OC = 8$. 如图, 过 E 点作 $EM \perp x$ 轴于点 M , 则在 $Rt\triangle AEM$ 中, $EM = OF = t - 2, AM = OA + OM = OA + EF = 4 + \frac{1}{2}t$, 由勾股定理得: $AE^2 = AM^2 + EM^2 = \left(4 + \frac{1}{2}t\right)^2 + (t - 2)^2$. 在 $Rt\triangle AEG$ 中, 由勾股定理得: $EG = \sqrt{AE^2 - AG^2} = \sqrt{\left(4 + \frac{1}{2}t\right)^2 + (t - 2)^2 - 8^2} = \sqrt{\frac{5}{4}t^2 - 44}$. 在 $Rt\triangle ECF$



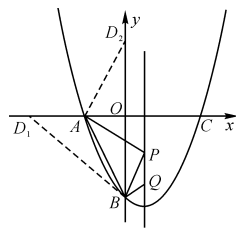
(第 22 题)

中, $EF = \frac{1}{2}t, CF = OC - OF = 10 - t, CE = CG + EG = 4 + \sqrt{\frac{5}{4}t^2 - 44}$. 所以 $EF^2 + CF^2 = CE^2$, 即 $\left(\frac{1}{2}t\right)^2 + (10 - t)^2 = \left(4 + \sqrt{\frac{5}{4}t^2 - 44}\right)^2$. 解得, $t_1 = 10$ (舍去), $t_2 = 6$. 所以 $t = 6$.

23. 解: (1) 由题意得: $A(-1, 0), B(0, -\sqrt{3}), C(2, 0)$. 当 $AP + BQ$ 最小时, 四边形 $ABQP$ 的周长最小. 点 B 向上平移 $\frac{\sqrt{3}}{3}$ 个单位得 B' 的坐标 $\left(-\frac{2\sqrt{3}}{3}, 0\right)$, $\therefore CB'$ 就是 $AP + BQ$ 的最小值, 即 $AP + BQ = CB' = \frac{4\sqrt{3}}{3}$. \therefore 四边形 $ABQP$ 周长的最小值是 $\frac{5\sqrt{3}}{3} + 2$;

(2) 如图 2, 当四边形 $ABQP$ 周长取最小值时, 得:

点 $P\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$, 点 $Q\left(\frac{1}{2}, -\frac{5\sqrt{3}}{6}\right)$, 又点 $B(0, -\sqrt{3})$, 得 $\angle BPQ = \angle OBP = 30^\circ, \angle OBQ = 60^\circ, \angle PBQ = \angle OBQ - \angle OBP = 30^\circ, \therefore BQ = PQ, \angle BQP = 120^\circ$. ① 当点 D_1 在直线 BA 的左侧时, $\triangle ABD_1 \sim \triangle QBP, \angle D_1AB = \angle PQB = 120^\circ, \therefore \angle OAB = 60^\circ, \therefore D_1$ 落在 x 轴上, 又 $D_1A = AB = 2. \therefore$ 点 D_1 坐标为 $(-3, 0)$. ② 当点 D_2 在直线 BA 的右侧时, 由 $\angle D_2BA = \angle D_1BA = 30^\circ, \therefore$ 点 D_2, D_1 关于直线 AB 对称, \therefore 点 D_2 落在 y 轴上, $D_2B = D_1B = 2\sqrt{3}. \therefore$ 点 D_2 的坐标为 $(0, \sqrt{3})$.



(第 23 题)

数学模拟试卷(十四)

一、1. A 2. C 3. B 4. D 5. D 6. C 7. D 8. B 9. C 10. A

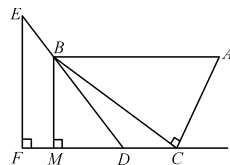
二、11. $a(a + \sqrt{2})(a - \sqrt{2})$ 12. -3 13. $\frac{84}{5}\pi$ 14. 29.5 20 15. $y = -\frac{4}{25}(x - 1)^2 + 4$ -8 16. 5

三、17. 解: 化简得 $\frac{1}{x-y}, x = \sqrt{3}, y = 1$, 原式 $= \frac{1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{2}$.

18. 解: (1) $\because \angle BAC = 24^\circ, CD \perp AB, \therefore \sin 24^\circ = \frac{CD}{AC}, \therefore CD = AC \sin 24^\circ = 30 \times 0.40 = 12\text{cm}, \therefore$ 支撑臂 CD 的长为

12cm; (2)过点C作 $CE \perp AB$ 于点E,当 $\angle BAC=12^\circ$ 时, $\therefore \sin 12^\circ = \frac{EC}{AC} = \frac{EC}{30}$, $\therefore EC=30\sin 12^\circ=30 \times 0.20=6\text{cm}$, $\therefore CD=12\text{cm}$, $\therefore ED=6\sqrt{3}\text{cm}$, $\therefore AE=\sqrt{30^2-6^2}=12\sqrt{6}\text{cm}$, $\therefore AD=(12\sqrt{6}+6\sqrt{3})\text{cm}$ 或 $AD=(12\sqrt{6}-6\sqrt{3})\text{cm}$.

19. 解:过点B作 $BM \perp FD$ 于点M.在 $\triangle ACB$ 中, $\angle ACB=90^\circ$, $\angle A=60^\circ$, $AC=10$, $\therefore \angle ABC=30^\circ$, $BC=AC \cdot \tan 60^\circ=10\sqrt{3}$, $\therefore AB \parallel CF$, $\therefore \angle BCM=30^\circ$. $\therefore BM=BC \cdot \sin 30^\circ=10\sqrt{3} \times \frac{1}{2}=5\sqrt{3}$, $MD=BM=5\sqrt{3}$, $CD=CM-MD=15-5\sqrt{3}$.



(第19题)

20. 解:(1)作图略;(2) $\because AE$ 是直径, $\therefore \angle ACE=90^\circ$, $\therefore \angle ADB=\angle ACE=90^\circ$, $\therefore \angle B=\angle E$, $\therefore \triangle ABD \sim \triangle ACE$, $\therefore \frac{AB}{AE} = \frac{AD}{AC}$, $\therefore \frac{5}{AE} = \frac{4}{6}$, $\therefore AE = \frac{15}{2}$, \therefore 外接圆的半径为 $\frac{15}{4}$.

21. (1)证明:设 EF 交 AD 于 G ,在 $\triangle AFG$ 和 $\triangle ABD$ 中,由 $FG \parallel BD$,得到 $\frac{FG}{BD} = \frac{AF}{AB}$,在 $\triangle AFE$ 和 $\triangle ABC$ 中,由 $FE \parallel BC$,得到 $\frac{AF}{AB} = \frac{FE}{BC}$,故 $\frac{FG}{BD} = \frac{FE}{BC}$,在 $\triangle EPF$ 和 $\triangle CPB$ 中,由 $FE \parallel BC$,得到 $\frac{PF}{PC} = \frac{FE}{BC}$,故 $\frac{FG}{BD} = \frac{PF}{PC}$,在 $\triangle FPG$ 和 $\triangle CPD$ 中,由 $FG \parallel DC$,得到 $\frac{FG}{DC} = \frac{PF}{PC}$,故 $\frac{FG}{BD} = \frac{FG}{DC}$,所以 $BD=CD$ (四个比例式得出1个得1分,全对并过渡正确得4分)

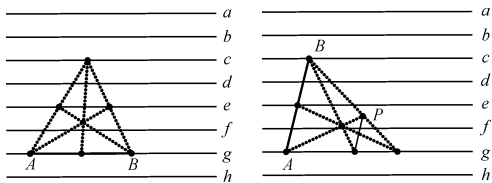


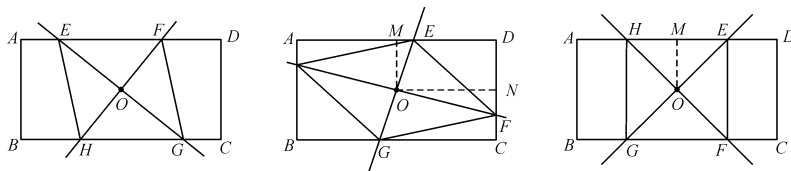
图3

图4

(2)作图,每题3分,共6分.

22. 解:(1)真命题;两定点 $(-1,0)$ $(-2,-2)$;(2)假命题; m 的最大整数值是 -2 ;(3)假命题;应该是 $y_1 < y_2$;(4)真命题; $EF=\sqrt{5}$.

23. 解:(1)四边形 $EFGH$ 是菱形, \because 矩形 $ABCD$ 是中心对称图形, O 是对称中心, $\therefore OE=OG,OF=OH$, \therefore 四边形 $EFGH$ 是平行四边形, $\because EG \perp FH$, \therefore 平行四边形 $EFGH$ 是菱形;



(2)①如图1,当 F 在边 AD 上时,作 $OM \perp AD$ 于 M ,则 $OM=1,EM=2-x$,由 $\triangle EMO \sim \triangle OMF$ 得: $\frac{MF}{OM} = \frac{OM}{EM}$,

$\therefore FM = \frac{1}{2-x}$, $\therefore EF = 2-x + \frac{1}{2-x}$, $\therefore S = 2 \left(2-x + \frac{1}{2-x} \right) = 4-2x + \frac{2}{2-x}$,此时 $0 \leq x \leq \frac{3}{2}$;

②如图2,当 F 在边 CD 上时,作 $OM \perp AD$ 于 M ,作 $ON \perp CD$ 于 N ,则 $OM=1,ON=2,EM=x-2$,由 $\triangle OME \sim \triangle ONF$ 得: $\frac{OF}{OE} = \frac{ON}{OM} = 2$, $\therefore OF = 2OE$ $\therefore S = \frac{1}{2} \cdot EG \cdot FH = 2 \cdot OE \cdot OF = 4 \cdot OE^2 = 4[1^2 + (x-2)^2] = 4(x-2)^2 + 4$,

此时 $\frac{3}{2} < x \leq \frac{5}{2}$,③如图3,当 F 在边 BC 上时,作 $OM \perp AD$ 于 M ,则 $OM=1,EM=x-2$,由 $\triangle EMO \sim \triangle OMH$

得: $MH = \frac{1}{x-2}$, $\therefore EH = x-2 + \frac{1}{x-2}$, $\therefore S = 2 \left(x-2 + \frac{1}{x-2} \right) = 2x-4 + \frac{2}{x-2}$,此时 $\frac{5}{2} < x \leq 4$;

(3)当 $0 \leq x \leq \frac{3}{2}$ 时,由 $2 \left(2-x + \frac{1}{2-x} \right) = \frac{1}{2} \times 4 \times 2$ 得: $x=1$.当 $\frac{3}{2} < x \leq \frac{5}{2}$ 时,由 $4(x-2)^2 + 4 = \frac{1}{2} \times 4 \times 2$ 得: $x=2$.当 $\frac{5}{2} < x \leq 4$ 时,由 $2 \left(x-2 + \frac{1}{x-2} \right) = \frac{1}{2} \times 4 \times 2$ 得: $x=3$.综上所述: x 的值为1或2或3.